PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 60 SEPTEMBER, 1934 No. 7

TECHNICAL PAPERS
DISCUSSIONS
APPLICATIONS FOR ADMISSION AND TRANSFER

Published monthly, except June and July, at 99–129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers, Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

Copyright, 1934, *by the* AMERICAN SOCIETY OF CIVIL ENGINEERS

Printed in the United States of America

CURRENT PAPERS AND DISCUSSIONS

		Discus clos
Progress Report of Special Committee on Earths and Foundations	May, 1933	
Discussion	Aug., Sept., Oct., Nov., Dec., 1933, Jan., Aug., 1934	Clos
Water Power Development of the St. Lawrence River. *Daniel W. Mead*	Aug., 1933	
Discussion	Aug., Nov., Dec., 1933	Clos
Water-Bearing Members of Articulated Buttress Dams. *Hakan D. Birke*	Sept., 1933	
Discussion	Feb., 1934	Clos
Some Soil Pressure Tests. *H. de B. Parsons*	Nov., 1933	
Discussion. (Author's closure)	Jan., Feb., Mar., Apr., May, Sept., 1934	Clos
Practical River Laboratory Hydraulics. *Herbert D. Vogel*	Nov., 1933	
Discussion	Feb., Mar., Apr., May, 1934	Clos
Formation of Floc by Ferric Coagulants. *Edward Bartow, A. P. Black, and Walter E. Sansbury*	Dec., 1933	
Discussion	Mar., Apr., 1934	Clos
Modifying the Physiographical Balance by Conservation Measures. *A. L. Sonderegger*	Dec., 1933	
Discussion	Mar., Apr., May, Aug., Sept., 1934	Sept.,
Model of Calderwood Arch Dam. *A. V. Karpov and R. L. Templin*	Dec., 1933	
Discussion	Apr., May, Sept., 1934	Sept.,
An Approach to Determinate Stream Flow. *Merrill M. Bernard*	Jan., 1934	
Discussion	Mar., Apr., May, Sept., 1934	Sept., 1
Discharge Formula and Tables for Sharp-Crested Suppressed Weirs. *C. G. Cline*	Jan., 1934	
Discussion	May, Sept., 1934	Sept., 1
Renewal of Miter-Gate Bearings, Panama Canal. *Clinton Morse*	Jan., 1934	
Discussion	May, 1934	Sept., 1
Loss of Head in Activated Sludge Aeration Channels. *Darwin Wadsworth Townsend*	Jan., 1934	
Discussion	Mar., May, 1934	Sept., 1
Williot Equations for Statically Indeterminate Structures in Combination with Moment Equations in Terms of Angular Displacements. *Charles A. Ellis*	Jan., 1934	
Discussion	Sept., 1934	Oct., 1
Rainfall Studies for New York, N. Y. *S. D. Bleich*	Feb., 1934	
Discussion	May, 1934	Oct., 1
Flexible "First-Story" Construction for Earthquake Resistance. *Norman B. Green*	Feb., 1934	
Discussion	May, Aug., 1934	Oct., 1
Investigation of Web Buckling in Steel Beams. *Inge Lyse and H. J. Godfrey.*	Feb., 1934	
Discussion	Aug., 1934	Oct., 1
Analysis of Sheet-Pile Bulkheads. *Paul Baumann*	Mar., 1934	
Discussion	May, Aug., 1934	Oct., 1
A Generalized Deflection Theory for Suspension Bridges. *D. B. Steinman.*	Mar., 1934	
Discussion	May, Aug., Sept., 1934	Oct., 1
Sand Mixtures and Sand Movement in Fluvial Models. *Hans Kramer*	Apr., 1934	
Discussion	Aug., Sept., 1934	Nov., 1
Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges. *Wilbur M. Wilson*	Apr., 1934	
Discussion	Aug., Sept., 1934	Nov., 1
The Reservoir as a Flood-Control Structure. *George R. Clemens*	May, 1934	Nov., 1
Stresses in Space Structures. *F. H. Constant*	May, 1934	
Discussion	Sept., 1934	Nov., 1
Experiments with Concrete in Torsion. *Paul Andersen*	May, 1934	
Discussion	Aug., 1934	Nov., 1
Wave Pressures on Sea-Walls and Breakwaters. *David A. Molitor*	May, 1934	
Discussion	Sept., 1934	Nov., 1
Flow of Water Around Bends in Pipes. *David L. Yarnell and Floyd A. Nagler.*	Aug., 1934	Nov., 1
Street Thoroughfares: A Symposium	Aug., 1934	Nov., 1
Eccentric Riveted Connections. *Eugene A. Dubin*	Aug., 1934	Nov., 1
Determination of Trapezoidal Profiles for Retaining Walls. *A. J. Sutton Pippard*	Aug., 1934	Nov., 1

CONTENTS FOR SEPTEMBER, 1934

PAPERS

PAGE

Security from Under-Seepage — Masonry Dams on Earth Foundations.
By E. W. Lane, M. Am. Soc. C. E. 929

Uplift and Seepage Under Dams on Sand.
By L. F. Harza, M. Am. Soc. C. E. 967

DISCUSSIONS

Some Soil Pressure Tests.
By H. de B. Parsons, M. Am. Soc. C. E. 1001

Modifying the Physiographical Balance by Conservation Measures.
By Messrs. J. B. Lippincott, and Rhodes E. Rule 1011

Model of Calderwood Arch Dam.
By Messrs. P. Wilhelm Werner, Eldred D. Smith, Elmer O. Bergman, and E. Probst. 1021

An Approach to Determinate Stream Flow.
By Messrs. C. H. Eiffert, and Charles S. Bennett 1037

Discharge Formula and Tables for Sharp-Crested Suppressed Weirs.
By W. Bruce McMillan, M. Am. Soc. C. E. 1040

Williot Equations for Statically Indeterminate Structures, in Combination with Moment Equations in Terms of Angular Displacements.
By Messrs. Ralph A. Tudor, Leon S. Moisseiff, and A. A. Eremin 1048

A Generalized Deflection Theory for Suspension Bridges.
By Messrs. L. J. Mensch, A. A. Eremin, Hans H. Bleich, F. H. Frankland, Gustav Lindenthal, Julian W. Shields, A. W. Fischer, and J. M. Frankland 1055

Sand Mixtures and Sand Movement in Fluvial Models.
By Messrs. R. H. Keays, and F. T. Mavis 1075

CONTENTS FOR SEPTEMBER, 1934 *(Continued)*

PAGE

Laboratory Tests of Multiple-Span Reinforced Concrete Arch Bridges.
By Carroll L. Mann, Jr., Esq. 1080

Stresses in Space Structures.
By Messrs. William R. Osgood, L. E. Grinter, and Charles M. Spofford 1085

Wave Pressures on Sea-Walls and Breakwaters.
By Charles E. Fowler, Esq. 1093

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

The Society is not responsible for any statement made or opinion expressed in its publications

---

MEMBERSHIP

Application for Admission and Transfer........following page 1094

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

SECURITY FROM UNDER-SEEPAGE MASONRY DAMS ON EARTH FOUNDATIONS

By E. W. Lane[1], M. Am. Soc. C. E.

Synopsis

This paper gives the results of an investigation of more than two hundred masonry dams with various kinds of earth foundation, to determine the length of the percolation path necessary to prevent failure from under-seepage or piping. Based on this study, a new method of analysis of such structures has been developed, which generally permits the use of smaller seepage distances than are ordinarily considered to be safe.

When a masonry dam is founded on earth, some of the water from the reservoir percolates beneath it and appears on the down-stream side. If the velocity of the flow where it emerges is sufficient, particles of the foundation material will be carried away by the water, and thus decrease the resistance to the percolation. The result is an increased velocity and greater erosion, ultimately causing the formation of a channel or "pipe" beneath the dam, which may enlarge rapidly and cause the failure of the structure. This process is known as piping. To prevent piping it is necessary to design the dam so that the velocity of the seepage water as it emerges on the down-stream side is insufficient to remove the foundation material. This is accomplished by making the route along which the water may percolate of such a length that the velocity of flow is reduced to a safe value. The investigation described herein was undertaken to determine from data on actual dams the length of the percolation path necessary to insure safety for dams founded on various kinds of material.

A more complete report, including drawings that show dimensions and typical sections of all the dams, as well as a comprehensive bibliography has been filed for research purposes in Engineering Societies Library, 29 West 39th Street, New York, N. Y.[1a]

Note.—Discussion on this paper will be closed in December, 1934, *Proceedings*.

[1] Research Engr., U. S. Bureau of Reclamation, Denver, Colo.

[1a] A lithographic set of the drawings may be purchased at a cost of 50 cents per copy from the Secretary of the Society.

THEORETICAL CONSIDERATIONS

The law of flow of water through earth was determined many years ag by H. Darcy[2]. Later work was done in the United States by the late Allen Hazen, M. Am. Soc. C. E.[3], and C. S. Slichter[4], and in India by Co J. Clibborn[5] and Mr. J. S. Beresford. Their experiments showed that (neglecting temperature effects) the discharge through a column of soil of a given composition varied directly with the head, directly with the cross-sectiona area, and inversely with the length. This relation, commonly known a Darcy's law, may be expressed as:

$$Q = c_1 \frac{H A}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1$$

in which, Q is the discharge, in cubic feet per second; H, the head, in feet A, the cross-sectional area, in square feet; L, the length, in feet; and c_1 a coefficient which depends on the character of the material.

Substituting for Q the value, $A\ V$, Equation (1) becomes,

$$L = c_1 \frac{H}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2$$

For a given class of material there is a definite maximum velocity, V_m, a which the water can emerge below the dam without carrying away the foundation material and causing the failure of the structure. Combining this value of V_m with c_1, which also depends on the material, to form a new coefficient

$c_2 = \frac{c_1}{V_m}$, the expression,

$$L_n = c_2\ H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

is obtained, in which, L_n is the minimum safe length of travel path and c_2 is a coefficient depending on the foundation material.

The foregoing theory seems to be generally accepted. In applying it, two difficulties arise: (1) How shall the length of the travel path, L_n, be measured and (2) what value of c_2 can safely be used for various classes of material? Before going further into a discussion of these points it may be well to give the history of the development of this phase of dam design.

HISTORICAL

The first rational basis for the design of masonry dams on earth foundations seems to have been developed in India, as a result of the investigations of Colonel Clibborn[5] and Mr. Beresford. Colonel Clibborn was at one time Executive Engineer in charge of the Rolhikhand Canal Division of the United Provinces, India, where there were a number of dams founded on light sand, which had often given trouble. After he became Principal of homason College, together with Mr. Beresford, he carried out a classical et of experiments on the law of flow through sand. From these experiments Ir. Beresford concluded that the Narora Weir on the Ganges River was nsafe, because of excessive upward pressure on the apron, and made a report o that effect. At the time no special trouble had been experienced with he dam, but as a result of this report pressure pipes were placed in the pron to indicate the upward pressure beneath it. The pressure indicated n these pipes confirmed Mr. Beresford's conventions and by coincidence, he next day after the readings were taken (March 30, 1898), the apron at nother part of the dam was blown up, resulting in a breach of the weir. The failure of an important structure, following so promptly after the declaration of its instability, profoundly impressed the engineers of the United Provinces; and the hydraulic gradient theory of design became generally ccepted there about 1898.

---

[2] Pub. in Paris, France, in 1858.

[3] Rept., Massachusetts State Board of Health, 1892.

[4] *Water Supply Papers Nos. 67 and 140*, U. S. Geological Survey.

[5] "Experiments on the Passage of Water Through Sand", Govt. of India, Central Printing Office, 1902.

The first edition of "Practical Design of Irrigation Works", by W. G. Bligh, appeared in 1907, in which the theory was advanced that the stability of a weir on a porous foundation depended on the weight of the structure and not on the ratio of the percolation distance to the head. In his second edition, published in 1910, Bligh admitted the fallacy of his original contentions and explained his well-known theory that the safety of masonry dams on earth foundations depends on the length of the percolation path, which is along the line of contact of the structure and its foundation.

About the same time, this conclusion was also reached independently by Mr. W. M. Griffith whose paper[6] has not received the recognition in the United States which its value justifies. Before it was published, Bligh's second edition appeared, in which he also proposed that idea. The same conclusion seems to have been reached about that time by the late William Wolcott Tefft, M. Am. Soc. C. E., but did not find its way into print. The widespread use of Bligh's book, together with the publication of other articles and books[7] by him, has led engineers generally to give the credit for the idea to Bligh.

In 1911, a paper appeared, by Mr. Arnold C. Koenig,[8] giving rules for the design of masonry dams on earth foundations, which contains a number of valuable ideas.

Little appeared in technical literature for many years after these publications to aid the engineer in the practical design of masonry dams on earth foundations, but a number of measurements of upward pressure on actual and model dams have been published from time to time. Recently, methods have been devised for determining the upward pressure and path of percolation for various conditions. One of these is that used by Charles Terzaghi, M. Am.

---

[6] "The Stability of Weir Foundations on Sand and Soil Subject to Hydrostatic Pressure," *Minutes of Proceedings*, Inst. C. E., Vol. 197, Pt. III, 1913–14, p. 221.

[7] "Dams and Weirs"; "Dams, Barrages and Weirs on Porous Foundations", *Engineering News*, December 29, 1910, p. 708, and January 12, 1911, p. 52; "Weirs on Porous Foundations and with Pervious Floors", *Engineering News*, April 13, 1911, p. 444; "Lessons from the Failure of a Weir and Sluices on Porous Foundations", *Engineering News*, February 6, 1913, p. 266; and "Irrigation Headworks Repair and Dam Failure", *Engineering News*, June 8, 1916, p. 1070.

[8] *Transactions*, Am. Soc. C. E., Vol. LXXIII (1911) p. 175.

Soc. C. E.[9] It seems to have been first used for dam foundations by Phillip Forchheimer[10], but Professor Terzaghi's paper appears to be the first explanation of this use in English.

By means of a flow net[11], the direction of the currents of the water and the pressure throughout the material beneath the dam can be computed for various assumed conditions. Another interesting method of obtaining the same information has been devised by Professor N. N. Pavlovsky[12], which makes use of the similarity of the laws of flow of the electric current through a conductor and the flow of water through soil. With the methods of either Professor Terzaghi or Professor Pavlovsky it is possible to obtain a rigid solution for the given assumptions, and they are very useful in securing picture of the flow of the water beneath the dam. As will be discussed later, however, the results obtained, if the ordinary assumptions are used, must be applied to actual cases with caution.

Four interesting and instructive papers have recently appeared, which throw much light on the problem. Two of these are papers presented at the Punjab Engineering Congress of 1930, by Mr. A. N. Khosla.[13] The third is a paper by Mr. S. Leliavsky.[14] The fourth is a paper recently presented by Mr. Khosla[15] before the Punjab Engineering Congress, giving the results of measurements of upward pressure on the Panjnad Weir.

Course of Seepage Beneath a Dam

The path that the water takes in flowing beneath a dam might be determined with reasonable accuracy by either the flow net, or by the electrical methods, if all the conditions governing the flow were known accurately. This is never the case, however, and for purposes of design certain assumptions must be made. In the second edition of his "Practical Design of Irrigation Works", Bligh proposes an analysis on the assumption that the water follows a path along the line of contact of the dam foundation (including the sheet-piling) with the foundation material. The same method was suggested by Mr. Griffith. This contact between the dam and the foundation material is sometimes called the line of creep, and the method may be called the line-of-creep method. Another method that has been advocated to some extent, may be called the short-path method, and is based on the assumption that the course taken by the percolating water is the shortest path through the pervious material between the head-water and the tail-water. Neither of these methods gives a true picture of the actual conditions of flow beneath dams, but they are useful methods of practical design. The line-of-reep method has gained a wide acceptance, and most masonry dams on earth oundations have been designed according to it. It has been used in the rrigation works of the United States Bureau of Reclamation for many years vith satisfactory results. The short-path theory has been but little used.

---

[9] *Technical Publication No. 215,* Am. Inst. of Mining and Metallurgical Engrs., p. 31

[10] "Hydraulics", by P. Forchheimer, 1924, p. 448.

[11] "Hydraulic Laboratory Practice", by the late John R. Freeman, Past-President and Hon. M., Am. Soc. C. E., Am. Soc. Mech. Engrs., 1929, p. 605.

[12] *Engineering News-Record,* Vol. 112, June 14, 1934, p. 765.

[13] "Hydraulic Gradients in Subsoil Water Flow in Relation to Stability of Structures Resting on Saturated Soils", *Paper No. 138;* and "Stability of Weirs and Canal Works: An Application of the New Theory of Hydraulic Gradient", *Paper No. 142,* Punjab Eng. Congress, 1930.

[14] "On Percolation under Aprons of Irrigation Works", by S. Leliavsky (published monthly).

[15] *Paper No. 162,* Punjab Eng. Congress.

In designing a masonry dam on an earth foundation it is customary to assume that the path of the water flowing beneath it is always normal to the axis of the dam. Analyses are made, therefore, of cross-sections of the dam perpendicular to the axis. If the material on which the structure is founded varies along its length, it may be necessary to investigate more than one section. The assumption of normal flow is sufficiently accurate for most conditions. In unusual cases, however, a more accurate analysis may be necessary. In analyzing the security of the dam at the abutments, normal flow cannot be assumed.

Comparison of Design Methods

Although the line-of-creep method has been widely used in dam design, it has been subject to some criticism. Part of this criticism is believed to be due to an improper presentation of the case in its favor. Bligh states that the water follows the line of creep and not the path of least resistance. This statement is believed to be in error, because the fact that water would take the path of least resistance seems almost axiomatic. If water flows along the line of creep instead of the shorter path directly through the foundation material it is because the resistance to travel along the line of creep is less than along the shorter path. That resistance along the line of creep may be less than through the foundation material seems quite reasonable, on account of the difficulty of securing as intimate a contact between the more or less plain surfaces and the foundation material as between the individual particles of the foundation material.

It should be remembered that the line-of-creep method is intended to give a dam safety at all points. This requires that it apply to the worst condition that will happen with every reasonable care in construction. The seepage may not follow the creep line at many cross-sections of the dam, but the points where there is most danger of failure are likely to be those where contact between the dam and the foundation material is not so close, and, therefore, where the line-of-creep method best applies.

The principal weakness of the line-of-creep theory is that it assumes the resistance to flow along all parts of the contact between the dam and the foundation to be the same. In practice, the contact between vertical and steeply sloping surfaces is more likely to be intimate than that along horizontal or slightly sloping surfaces. In a masonry dam on earth foundations there is likely to be unequal settlement which will cause less pressure at some points than at others, in places, even a slight lifting up of the masonry from its contact with the earth beneath. The earth beneath a dam may not be compact, and may settle after the dam is built, leaving void spaces beneath the floor, especially where the dam is founded on piles. This action is sometimes called "roofing".

Another cause of roofing is given by J. C. Oakes[16], M. Am. Soc. C. E as follows:

“If the bed of the foundation [of a dam] is below the water level, ar pumping is required, there will be a small space between the base of t masonry and the sand, caused by erosion by the flow of water from und one block of masonry while the next one is being placed. In other words t structure is ordinarily supported wholly by the piles, with almost a certaint that there will be a space between the sand and the masonry. Under suc circumstances, any upward pressure that develops under the dam will b uniform from the sheet-piling to the toe, and will depend on the tightne of the sheeting and the ease of escape below the dam, * * *.

“* * * the writer has watched the placing of concrete, and in no case ha he found, on the works under discussion [Ohio River Dams, Nos. 43 and 4 that the concrete rested on the sand after sufficient time had been given to set. The bed of the foundation of the various parts of these is abou 10 ft below water. Construction has been carried on within coffer-dam and during stages of the river from low water to 14 ft above it. Owing to t permeable nature of the material, there has always been considerable percol tion, which has required pumping to keep the pit sufficiently clear of wate to enable construction to proceed. The water escaping from under the concret already placed, carries away the fine material directly under the concrete, lea ing a space between it and the sand through which the transmission c pressure will be direct, and, consequently, any pressure which may be develope will be uniformly exerted over the whole base of the structure.”

A study of the results of upward pressure measurements on actual da discloses much evidence to support Colonel Oakes’ conclusions.

On vertical or steeply sloping faces, roofing will not occur because the voi spaces, if they should be formed, would be filled again by the inability of th earth to maintain the steep slope; the earth from the steep bank would mo down and fill the void. For this reason the contact of the foundation wit such surfaces is close and offers more security against piping than contac beneath horizontal surfaces.

D. C. Henny, M. Am. Soc. C. E., has suggested that the vertical creep ma be more effective than the horizontal creep because the stream-bed materia is likely to be laid down in nearly horizontal layers or lenses of varyin permeability. By building cut-offs through these layers, the flow through th relatively permeable ones is stopped, and the water is forced either to pas through the more impermeable layers or to seek a much longer route, wit consequent reduction of velocity.

Piping failure, therefore, should be considered as possible from two large independent causes: (1) Direct percolation through the foundation materia itself; and (2) percolation along the contact of the dam and sheet-piling with the foundation material. Considerable light on the first of these causes ha been obtained by experiment, but Cause (2) can only be evaluated by investi gating a large number of structures. A weakness of the ordinary line-of creep theory is that it considers only the second of these causes. Anothe weakness is that it is possible to drive lines of sheet-piling so close togethe that the short path may be so small that failure can occur by flow which will not follow the path of creep at all.

---

[16] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 469–470.

The weakness of the short-path theory is that it takes no account of the greater probability of percolation along the line of contact of the structure and its foundation. In cases where the lines of sheet-piling are very close together, however, it may be more reliable than the creep theory. The short-path principle would seem to be obviously inferior to the line-of-creep analysis as applied to clay or hardpan foundation, since the foundation material in this case would be so nearly impermeable that the seepage along the line of creep would be much greater. Another difficulty with the short-path analysis is that it gives no method of estimating the magnitude or distribution of upward pressure beneath the dam. As some estimate of this pressure is required in order to determine the required thickness of the apron, additional assumptions are necessary if the dam is to be designed by the short-path theory throughout.

The flow net and electric analogy methods are essentially the same. Both should give the same results for the same assumptions. Professor Terzaghi[9] does not take into consideration the greater probability of percolation along the line of contact. Whether or not Professor Pavlovsky[12] does, is not known. It would be possible to do it by either method, however, by assuming the relative permeability along this line as compared with that through the foundation material. The weakness of both methods is the necessity of a detailed knowledge of sub-surface conditions and the lack of data relating the results obtained, as shown by the flow net, to the safe limits for the various classes of material.

Nevertheless, the flow net and electric analogy methods may prove to be useful tools in analyzing unusual conditions, and in forming a mental picture of what takes place under certain conditions. It is hoped that further studies along this line will be made to clear up some of the problems of design. There is a great opportunity for useful experimentation in this field. The results that would be obtained by the flow net or by the electrical analysis would roughly correspond in a homogeneous medium to that obtained by the short-path method. Both are based on the consideration of flow directly through the foundation material. The short-path method may be considered, therefore, as a rough approximation of the flow net, or electrical analysis.

A weakness of all the methods, as ordinarily applied, is that the flow is considered as taking place only in a single plane. This is not necessarily the case however, because, as Mr. Griffith has pointed out,[17] when a pipe tends to form, it provides a line of lowered pressure, and water from both sides flows toward it, as well as that in the plane of the incipient "pipe". Analyzing the stability of a dam by means of its cross-section is a useful device, but one is likely to be blinded to the true conditions by thinking too rigidly in terms of cross-section only.

REQUIRED LENGTH OF PERCOLATION PATH

Not only have there been differences of opinion regarding the path along which the percolation should be assumed to occur, but this is true also regarding the length of the path necessary to insure safety from piping failure.

---

[17] *Minutes of Proceedings*, Inst. C. E., Vol. 197, 1913–14, Pt. III, p. 223.

As the result of his study of dams and dam failures, Bligh arrived a values of the creep-head ratio which he believed would make dams safe from piping failure. The description of the various classes of material and th values of the ratios varies somewhat in different publications by Bligh The following list gives the classification in his latest publication:[18]

	Safe ratio
River beds of light silt or sand of which 60% passes the 100-mesh sieve, as those of the Nile or the Mississippi Rivers	18
Fine micaceous sand of which 80% passes a 75-mesh sieve, as in Himalayan rivers and in such rivers as the Colorado	15
Coarse-grained sands, as in Central and South India	12
Boulders or shingle and gravel and sand mixed	5 to 9

Just how extensive a study of dams was made by Bligh as a basis for his data is not known, but the amount of material published is quite meager. It includes only two dam failures, both on fine sand foundations; and in one or both of these cases, unsound conclusions were drawn. There were no dam failures on silt, coarse sand, gravel, or boulders. In a later publication, Bligh discusses the failure of a head-gate of the Southern Alberta Land and Irrigation Company. The published data on this structure are so conflicting as to throw serious doubt on the reliability of his conclusion.[19] These statements are not made to disparage Bligh's work, which has formed the basis of the design of scores of safe dams, but merely to indicate that, apparently, it was based on meager data and that with much more extensive data, and better construction materials, a reduction of Bligh's values of C might be made without conflicting with well-established facts.

In an abridged paper,[6] Mr. Griffith gave the following values of the ratio of creep distance to head which "had been found sufficient to ensure stability" in the United Provinces of India:

Material	Limiting safe value of C
Fine micaceous sand	14½ to 16
Fine quartz sand	12½ to 14
Coarse quartz sand	10 to 12
Shingle	8
Boulders	4

Concerning shingles and boulders, Mr. Griffith states that "the question of loss by leakage may make higher values of $\frac{L}{H}$ advisable in these cases."

In an unpublished part of this paper he suggested "a 20% reduction in the values given where reliable vertical staunching of 10-foot depth was used."

In the third edition of Bligh's "Practical Design of Irrigation Works", which was revised and brought up to date by F. F. Woods, Chief Engineer of Irrigation Works, Punjab, India, Mr. Wood contends[20] that Bligh should have used a ratio of 11 for ordinary sand instead of 15. It will thus be seen that both Messrs. Griffith and Woods advocate somewhat lower values of C than Bligh. Independently, the writer arrived at the same conclusion from a study of the data on most of the dams included in the tables of this paper.

---

[18] "Dams and Weirs", 1916, p. 155.

[19] *Engineering Record*, Vol. 63, p. 589; Vol. 66, p. 376; and Engineering News, Vol. 69, p. 266; and Vol. 75, p. 1070.

A New Method of Analysis

As has already been pointed out, the existing methods of analysis are open to serious objections. The commonly used method, as advocated by Bligh, does not consider the greater resistance to flow along vertical contacts as compared with horizontal ones. The short-path method and the more exact flow net and electrical methods do not consider the lesser resistance along the contact of the masonry and foundation material as compared with that directly through the foundation material. All these methods have elements of truth, but all have weaknesses. A method should be devised which will combine the virtues of both without including their faults.

In the present state of knowledge the only method of analyzing the probability of failure from flow along the creep line seems to be a study of the action of actual dams. No exact data have yet been presented to show the relative resistance to the flow along the contacts as compared with that through the foundation material. It will be very difficult to obtain these data because points where danger of flow along the contact is great, will only occur, occasionally, and would probably be discovered only by extensive observations. The flow through the foundation material can be obtained with more exactness by experimental methods; but even in that case the unknown conditions of the foundations and the difficulty in determining the safety of the structure (even if the velocity of flow is known) make this method difficult to apply. It seems obvious, therefore, that while research along this line should be given every encouragement, the main reliance in dam design, for a long time in the future, must be on a somewhat empirical basis.

As the studies made in connection with this paper indicated faults of the existing methods of analysis, as well as the insufficiency of the factors used in the ordinary methods, the writer has attempted to develop a more rational method of design, as well as to establish more accurate coefficients. In order to devise a method that could readily be used by the designing engineer in planning actual structures, in addition to the aforementioned reason, the method developed was necessarily a somewhat empirical one.

For clarity in presenting the data on which writer's conclusions are based, it is necessary here to review, briefly, the basic facts of the new method developed.

From a study of all the available data it appeared that there were two distinct forms of piping, one in which the water passed along the line of contact of the structure and its foundation, as assumed in the Bligh theory, and the other in which it passed directly through the voids in the foundation material. In the former path the contact of the foundation on vertical or steeply inclined surfaces can be relied upon to offer more resistance to flow than along horizontal or slightly sloping contacts. In computing the safety of a structure, therefore, from this type of failure, the creep distances along horizontal or slightly sloping surfaces should be given less weight than those along vertical or steeply inclined surfaces. This method of estimating the stability of a structure may be called the weighted-creep method. As a result of these studies a weight of one-third is given to the horizontal or slightly inclined creep as compared with the other section of the path. For example, if a dam sustains a head of 10 ft and has a creep distance along horizontal or slightly inclined surfaces of 60 ft and along vertical and steeply inclined surfaces of 10 ft, the weighted creep distance would be $60 \times \frac{1}{3} + 10$, or 30 ft, and the weighted-creep-head ratio would be $30 \div 10 = 3.0$.

---

[20] See "Preface", p. VI.

For brevity in the remainder of this paper, creep along vertical surfaces or surfaces sloping more than 45° with the horizontal will be called "vertical creep", and other surfaces, "horizontal creep." It should be noted that "vertical or steeply inclined" refers to the position of the surface against which creep takes place, and not to the direction of the creep, which sometimes is not in the same direction as the inclination of the surface. The weighted-creep distance is the vertical creep plus one-third the horizontal creep, which, therefore, is practically always less than the creep distance.

As the water follows the line of least resistance, if the resistance to flow along the creep line is much greater than directly through the voids in the foundation material, much more water may take the latter course and failure from piping from this cause may result. The short path is a rough measure of the resistance through the voids. Both the weighted-creep and the short-path ratios, therefore, must be greater than their respective critical values for the type of material on which the dam is founded.

METHOD OF ANALYZING DATA ON EXISTING STRUCTURES

In this study an intensive search was made in all available engineering literature and other sources of information. All masonry or concrete dams on earth foundations were analyzed when the data were sufficiently complete. In many cases there is considerable uncertainty. When more than one description was found of a single structure there was often a surprising lack of agreement between them.

Frequently, there is considerable room for the exercise of judgment as to just where the travel path should be assumed to end. The travel path of the water was ordinarily assumed to end at its junction with the rip-rap down stream from the dam or at a reverse filter. If wooden cribs or loose or articulated concrete blocks were used, it was assumed to end at the up-stream edge of the rip-rap or filter. Although it is true that rip-rap, cribbing, or blocks may assist in preventing piping or a blow-out, it is believed that less uncertainty is introduced by considering that they do not add to the travel path than to assume that they do.

Some discrepancies were found in checking over the data given by Bligh with those available from other sources. When the data on some dams indicated that a certain creep-head ratio had been used in design, it was usually not possible to check this exactly from the dimensions of the structure. In order that all the data might be on the same basis, the same method was used throughout, although this might not give the same ratio that the designer of the dam believed he was using. In all cases the writer's best judgment was used in determining the most probable values for use in this paper. He would appreciate it if readers would call his attention to any cases in which his judgment may have been in error. At several points in the analysis of some of the dams it was necessary to make assumptions. One of these was in the case of rows of sheet-piling close together.

In computing the weighted-creep distance it was necessary to determine the division point between steeply sloping and slightly sloping contacts. This division should be at the steepest slope at which, unquestionably, a bank of earth under water would be unstable; that is, at which the bank undoubtedly would be certain to slip. This was assumed to be on a 1 on 1 slope. Any contact with a slope of 1 on 1, or steeper, was considered to be a "vertical" contact and, at a flatter slope, a horizontal contact. In a few cases a puddle or earth-fill against a dam rested on top of a part of the dam masonry. Although this was a horizontal contact, it was assumed to be a "vertical" contact, because the earth would press as closely on the masonry under these conditions as on a vertical surface. There is some evidence to indicate that the contact of puddle with earth is so close that it would be equivalent to vertical creep. In order to be conservative, however, this assumption was not made in computing the ratios used in this paper.

In several cases dams have been built with a filling of dry or broken stone beneath them. This would offer much less resistance to seepage than the contact between solid masonry and foundation material. Where the floor above this stone filling has not been vented, to allow the water to escape, the resistance of the creep line along this section has been assumed to be one-half as great as for solid masonry. This is a conservative estimate from the standpoint of this study, but would not be conservative if used in design. The use of such filling seems to be confined to old dams and is a bad practice. In one case it is believed to have contributed to the failure of a structure. In the case of a number of dams of the buttressed concrete type, no floor was provided between the buttresses, and the water passing beneath the up-stream cut-off could rise in this space and escape through vents provided in the down-stream face of the dam. If these vents were closed, the water would have to pass beneath another section of the dam to reach the tail-water. For dams of this type, the distances and ratios for both conditions have been computed although the shorter creep distance is probably the effective one.

Considerable uncertainty has been introduced in the determination of the creep distance by the presence of weep-holes or vents. In determining the distances used in this paper, weep-holes have not been considered as reducing the creep distance, but a separate analysis of them has been made.

There was sometimes uncertainty as to the head to use in estimating the ratios. In ordinary over-fall dams it was measured from the crest of the dam to tail-water elevation, or, if the latter was not given, to the stream bed below the dam. If crest gates or flash-boards were used, it was measured from their tops. The ordinary operating head was used, although, in some cases (especially in movable navigation dams), the structures, no doubt, were subjected occasionally to greater heads.

Several cases have been found in which dams were built on a layer of porous material, such as gravel or sand, which was underlaid by an impervious layer of clay or hardpan, into which the cut-off walls or sheet-piling extended. These cases are not subject to analysis in the ordinary way and, therefore, the data on them are given without analysis. Masonry dams on earth in which the cut-off was carried to solid rock were not analyzed.

CLASSIFICATION OF FOUNDATION MATERIALS

There is a great need of a more accurate and scientific classification of foundation materials than the common terms, gravel, coarse sand, etc. Several classifications on the basis of grain size have been made, but none is entirely satisfactory. One developed by the United States Bureau of Soils[21] has been used extensively in the classification of soils and of materials for earth dams. For dam foundation materials, however, it is objectionable because in its classifications, fine sand, coarse sand, etc., are finer material than the engineer usually has in mind when using these terms. Mechanical analyses of foundation materials could be obtained for only a few dams.

RESULTS OF ANALYSIS OF EXISTING STRUCTURES

The results of the analysis of the weighted-creep relations for all structures for which sufficient data could be obtained, are given in Table 1. Some of the description in Column (9) of this table can be further elaborated, as follows:

Table 1(a).—Dam No. 12.—A firm, impermeable, red clay, with at least 50% boulders.

Table 1(a).—Dam No. 20.—A very compact and unyielding gravelly hardpan.

Table 1(a).—Dam No. 27.—A part of this dam was situated on clay overlaid by gravel and sand.

Table 1(a).—Dam No. 28.—A part of this dam was situated on clay overlaid by a thin layer of gravel.

Table 1(b).—Dam No. 3.—A layer of gravel and fine material, underlaid by an impervious layer which, in turn, was underlaid by pervious fine material and boulders.

Table 1(b).—Dam No. 44.—From sand with a small percentage of gravel at the surface to heavy boulders with some sand and gravel at the bottom of the sheet-piles.

Table 1(c).—Dam No. 17.—Graded from silt and fine sand at the surface to coarse sand at the bottom of the sheet-piling.

---

[21] "Grouping of Soils on the Basis of Mechanical Analysis", by R. O. E. Davis and H. H. Bennett, *Circular No. 419*, U. S. Dept. of Agriculture.

Table 1(d).—Dam No. 33.—Fine sand at the surface, grading to coarse sand below.

Table 1(d).—Dam No. 37.—Beneath most of this dam there was a layer of clay; but there was a running layer of sand and mud below part of it.

The source of this information is given in detail form in the original manuscript filed in Engineering Societies Library.

It is probable that the larger structures in the United States are still giving reasonably satisfactory results or a record of the failure would have been published. The smaller structures in this country and those in foreign countries are probably still in use, although there is a slight chance that they may have failed.

Most of the failures are dams of poor design, but they serve to show approximately the limits of good design. Well-designed dams rarely fail, but more lessons can be learned from failures than from successes. Some well-known failures are not included, because the data are insufficient or too conflicting to permit reliable conclusions. These include the Hauser Lake Dam, Missouri River, the Grand Barrage on the Nile, and the head-gate of the Southern Alberta Land and Irrigation Company.

Several cases were found in which the data were so complex that they were difficult to analyze. This was the case in the Alcona and Upper Alameda Creek Dams; the dam in the Scioto River, at Columbus, Ohio; that on the Guadalupe River, in Texas; and the Sacramento, Calif., Weir. Additional information as to the reliability of the data on the various structures, the length of the period of service, and other pertinent facts, is given in the original record of this research, previously mentioned.

IMPORTANT FACTS SHOWN BY EXPERIENCE WITH EXISTING DAMS

A study of all the available records of percolation distances of existing masonry dams on earth foundations, and of those that have failed from piping, brings out three important facts: (1) Several dams have failed from piping with percolation distances which, judged by the ordinary standards, should be safe; (2) many dams have stood successfully with percolation distances much less than those previously recommended; and (3) the dams that failed had very little of their creep paths along vertical or steeply sloping surfaces, while those that stood, with much smaller distances, had a considerable proportion of such creep.

Failures have occurred with creep distances which ordinary standards would consider safe in the case of the Kulli Bye-Wash (Table 1(*d*), Dam No. 10), and the Deoha Barrage (Table 1(*d*), Dam No. 4). The first of these structures was founded on fine sand and failed with a creep-head ratio of between 14 and 16. For fine sand, Bligh recommends a ratio of 15 and Griffith, 14½ to 18 for fine micaceous sand, and 12½ to 14 for fine quartz sand. The Deoha Barrage on fine sand was threatened with a creep-head ratio of 17 and, finally, failed with a piping path with a length at least 26.6 times the head. None of these structures had vertical cut-offs.

TABLE 1.—Data for Analysis of Weighted-Creep Relations

Dam No. (1)	Name (2)	Location (3)	Head, in feet (4)	Creep Distance, in Feet: Vertical (5)	Creep Distance, in Feet: Horizontal (6)	Weighted Creep, $\frac{H}{3}$: Distance, in feet (7)	Weighted Creep, $\frac{H}{3}$: Ratio (8)	Foundati materia (9)
		(a) Dams on Clay and Hardpan						
	Failures:							
1	Woodward	Flanders Brook, Hill, N. H.	30	24	8	27	0.9	Hard mate probably pan
2	Dolgarrog	L. Eigian, N. Wales	10	3	15	8	0.8	Hard blue boulders
3	Fergus Falls	Red River, Minn	23	17	28	26	1.1	Hardpan
	*Non-Failures:**							
10	French Landing Plant	Huron, Mich	32	50	54	68	2.1	Clay
			32	50	8	53	1.6	....
11	Dolgarrog	L. Eigian, N. Wales	10	13	15	18	1.8	Hard blue boulders
12	Marysville	Washwaak River, N. B.	22	24	68	47	2.1	Red clay; boulders
13	Fergus Falls	Red River, Minn	23	70	34	82	3.6	Hardpan
14	Kettle Creek	Kettle Creek, Ont	20.5	18	48	34	1.7	Hard blue cl
15	Shataipore Weir	Vellar River, India	9	27	44	42	4.6	Clay
16	Rupar Weir	Sutlej River, India	11.5	18	100	51	4.5	Clay with ders
17	Cooke	Au Sable R., Mich	40	76	289	172	4.3	Clay
18	Sangum Anicut	Pennen, India	9.0	52	98	85.0	9.4	Sand on clay
19	Shawano	Wolf River, Wis	15	57	81	84	5.6	Stiff greasy c
20	Whiting St.	Holyoke, Mass	18	20	17	26	1.5	Gravelly har pan
21	N. Diversion	Orland Project, Calif.	4.7	......	5	2	0.4	Red clay; gr
22	Big Coulee	Big Coulee R., Mont	6	13	15	18	3.0	Clay
23	Sidhnai	Ravi River, India	7.5	35	46	50	6.7	Clay
24	Dam No. 2	Ouashita River, La	14.4	63	31	73	5.1	Dense clay
25	Dam No. 5	Ouashita River, La	6.9	77	34	88	12.8	Hard clay
26	Dam No. 8	Ouashita River, Ark	10.5	41	22	48	4.6	Mostly clay; s sand
			13.5	97	34	108	8.0	
27	Dam No. 5	Mohawk River, N. Y	15	63	77	89	5.9	On clay
28	Dam No. 6	Mohawk River, N. Y	15	63	77	89	5.9	On clay
29	Junction	Manistee River, Mich	50	199	165	254	5.1	Hard clay
			50	205	225	280	5.6	....
30	Hodenphyl	Manistee River, Mich	65	144	381	271	4.2	Clay
31	L. Washington	Seattle, Wash	25	60	190	123	5.0	Hard blue cl
32	Acatlan	Mexico	17.7	51	90	81	4.6	Compact cla
33	Aganoa River	Aganoa, P. I.	9.4	67	40	81.0	8.6	Blue clay sand
34	Santa Barbara	Philippine Islands	9.9	54	48	70	7.0	Clay
35	Cascade Plant	Thornapple, Mich	28	72	93.0	103	3.7	Tough grey c
36	Edenville Plant	Tittabawassee, Mich	45	47	123	88	2.0	Clay-gravel hardpan
			45	62	124	103	2.3	Hardpan hard)
37	Riley Plant	St. Joseph, Mich	14	48.5	45.0	63.5	4.5	Clay and gra
38	Sanford Plant	Tittabawassee, Mich	27.5	34.0	68	56	2.1	Clay-gravel hardpan
			27.5	42.0	68	65	2.4	Clay, gravel hardpan
39	Secords Plant	Tittabawassee, Mich	48	59	116	98	2.1	Clay-gravel hardpan
40	Smallwood Plant	Tittabawassee, Mich	27.5	38	84	66	2.4	Clay-gravel hardpan
			27.5	27.0	68	50	1.8	Clay, gravel, hardpan
41	Tobacco Plant	Tobacco, Mich	45	38.0	128	81	1.8	Clay-gravel hardpan
42	Barton Dam	Huron, Mich	25	40	53	58	2.3	Gravel and h pan
			25	40	8	43	1.7	....
43	Black River	Black R., Mich	15	58	52	75	5.0	Clay
			15	32	3	33	2.2	....
44	Built by	Des Moines Elec. Co., Iowa	8	18	4	19	2.4	Clay.
			8	22	33	33	4.1	Clay
45	For Sturgis	St. Joseph, Mich	25	12	8	15	0.6	Hardpan
46	Bassano	Alberta, Canada	48	34	165	89	1.8	Clay over sa

* No failures recorded.

TABLE 1.—(*Continued*)

m o.)	Name (2)	Location (3)	Head, in feet (4)	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material (9)
				Vertical (5)	Horizontal (6)	Distance, in feet (7)	Ratio (8)	
	*Non-Failures:**							
47	Lake St. Francis	St. Francis R., Que	37	46	48	62	1.7	Stiff clay with small boulders and gravel
			37	53	68	76	2.1	
48	Vandalia	Milk River, Montana	26	113	125	155	6.0	Stiff clay and clay with fine gravel
49	Upper Grays	Black Creek, Utica, N. Y.	30	45	110	82	2.7	Good clay, underlaid by hardpan
50	Scranton	Scranton, Pa	15.5	31	12	35	2.3	Sand and yellow clay
51	Baltic Main	Baltic, Conn	28.7	28	40	41	1.4	Clay and gravel
52	Baltic Side	Baltic, Conn	13.7	16	20	23	1.7	Clay and gravel
53	Batavia	Towanda Cr., N. Y	10	16	35	28	2.8	Clay and gravel
54	Bethlehem	L. Ammanoosuc, N. H	23	100	56	119	5.2	Boulders, sand, clay
55	Burnham	Burnham, Me	32	68	105	103	3.2	Clay
56	Kearsley Cr	Flint, Mich	25	106	138	152	6.1	Clay
57	Logan	Logan River, Utah	30	24	56	43	1.4	Sand and gravel
58	Mayfield R	Mayfield, N. Y	30	50	105	85	2.8	Hardpan
59	Massillon	Nimisilla Cr., Ohio	25.5	43	59	63	2.5	Clay and sand
60	New London	Conn	24	26	31.5	37	1.5	Clay and gravel
61	Newport	Sugar R., Nwpt., N. H	16.5	42	56	61	3.7	Clay with gravel
62	Patillas	Guayama, Porto Rico	5	16	25	24	4.9	Clay and marl
63	Prince Albert	Sask. R., Sask., Can	23	50	125	92	4.0	Gravel and clay
64	Somerville	New Jersey	7.5	27	13	31	4.2	Clay and coarse sand
65	Reservoir	St. Paul, Minn	25	10	25	18.3	0.7	Clay, 50% sand, gravel
66	Wakeman	Vermillion R., Ohio	15	30	42	44	2.9	Clay and shale
67	Wonder Lake	Nippersink Cr., Ill	20	25	68	48	2.4	Clay over sand; gravel
68	Hightstown	Rocky Brook, N. J	11	30	28	39	3.6	Clay, sand and silt
69	Pittsfield No. 2	Millbrook R., Mass	28.7	30	38	42.7	1.5	Clay and gravel
70	Stoney River	Dobbin, W. Va	51	85	60	105	2.1	Boulders, clay, gravel
71	Rodriguez	Tijuana, Mexico	180	300	256	385.3	2.1	Soft clay
72	Swift Current Dam	Sask., Canada	19	63	63	84	4.4	Clay
73	Gonzalez	Texas	18	52	53	70	3.9	Clay
74	Williamson	Sandy Cr., Cisco, Tex	63	72	45	87	1.4	Clay
75	Portland Dam	Portland, Mich	11	12	38	24.7	2.3	Clay
76	Foote Dam	Au Sable River Oscada, Mich	31.5	100	118	139.3	4.4	Clay
77	Five Channels		26	50	150	100	2.8	Clay
78	Loud Dam		34	53	160	106.3	4.4	Clay
79	Stronach	Manistee River, Wellston, Mich	20	37	30	47.0	2.3	Clay
80	Springfield	Springfield, Vt	15	28	23	36	2.4	Clay
81	Quaro Dam	Texas	30	40	35	51.6	1.7	Clay
82	Redlands	Gunnison R., Colo	12	50	57	69	5.7	Clay and gravel
83	Coleman Br	Chester, N. J	9	26	26	34.7	4.0	Boulders and clay soil
84	Springbank	London, Ont	20	70	34	81	4.0	Dense clay
85	Delhi Weir	Sone River, India	8	26	112	64	8.0	Clay and sand

(*b*) Dams on Gravel, Cobbles, and Boulders

	Failures:							
1	Puentes	Gwadalantin R., Spain	143	44	157	96.3	0.67	Sand and gravel; classification uncertain
2	Port Angeles	Elwha River, Wash	79	72	97	104	1.3	Gravel, and coarse sand
3	Pittsfield	West Brook, Mass	33	29	47	45	1.4	Gravel, fines, boulders
4	Coon Rapids	Miss. R., Minn	48	35	127	46	0.95	Sand and boulders
5	Plattsburg	New York	34	44	45	59	1.75	....

---

* No failures recorded.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Failures:							
6	At Durance	Durance R., France	22	11.2	44.6	50.2	2.3	Founded gravel
7	Stoney River	Stoney River, W. Va.	39	90	126	132	3.9	Boulders, cl gravel
8	Retaining wall	Ansonia Canal, Conn	18	24	36	36	2.0	Founded gravel w horizont creep in silt t
	*Non-Failures:**							
11	Dam No. 5	Ohio River, Penn	7.5	43	32	54	7.2	Gravel
12	Dam No. 10	Ohio R., Ohio-W. Va.	8.4	49	35	61	7.2	Gravel
13	Dam No. 6	Ouachita R., Ark	6.2	47	33	58	9.4	Gravel with fi sand
14	Dam No. 45	Ohio R., Ind.-Ky	9.0	87	35	99	11.0	Sand and sm gravel
15	Dam No. 46	Ohio R., Ind.-Ky	9.0	87	35	99	11.0	Sand and sm gravel
16	Dam No. 51	Ohio R., Ill.-Ky	8.0	87	35	99	12.3	Fine sand a gravel
17	Dam No. 35	Ohio R., Ohio-Ky	6.4	47	35	59	9.1	Sand and gra
18	Dam No. 36	Ohio R., Ohio-Ky	7.9	84	35	96	12.0	Sand and gra
			7.9	89	80	116	14.6	Sand and gra
			7.9	80	20	87	10.9	Sand and gra
19	Dam No. 43	Ohio R., Ind-Ky	9.0	86	40	99	11.0	Sand and gra
20	Dam No. 52	Ohio R., Ill-Ky	12.0	85	40	99	8.2	Sand, gravel, l some clay
21	Dam No. 53	Ohio R., Ill-Ky	13.4	87	35	99	7.4	Sand, gravel, l some clay
22	Pinhook	Maquoketa R., Iowa	25	68	61.5	89	3.5	....
23	Richmond	Indiana	5	11	16	16.3	3.3	Gravel and san
24	Power Intake	Salt River, Ariz	6.5	43	18	49	7.5	Gravel and san
25	Island Park	Miami River, Ohio	7.5	37	29	47	6.2	....
26	Puentes	Spain (see No. 1)	75	51	288	147	2.0	
27	Miller Cr	Miller Creek, Ore	5.0	14	14	19	3.8	Sand, gravel, an earth
28	Sardorab	Arax R., Russia	16.7	57	108	93	5.5	Coarse sand an gravel
29	Cherreoksky	Arpachay R., Armenia	13.3	35	59	55	4.1	Coarse sand an gravel
30	Limbourg	Meuse R., Holland	12	84	131	128	10.7	Coarse gravel and sand
31	Herr Island	Allegheny R., Penn	7	45	48	61	8.7	Gravel
32	Mirowitz	Czecho-slovakia	12.8	60	33	71	5.5	Gravel
33	Hamilton	Miami, Ohio	7.5	31	31	41.3	5.5	Gravel
34	Fremont	Sacramento R., Calif	3.5	21	35	33	9.4	Gravel
35	Dam No. 7	Mohawk R., N. Y	12	32	63	53	4.4	Gravel
36	Dam No. 13	Mohawk R., N. Y	8	24	52	41.3	5.2	Gravel
37	Horse Creek	Horse Creek, Wyo	6.4	18	17	24	3.8	Coarse sand an gravel
38	Mesilla	Rio Grande, N. Mex	8.7	24	40	37.3	4.3	Medium san with fa sprinkling o gravel
39	Mandalay	Madaya R., Burma	11.3	10	78	36	3.2	Gravel an shingle
40	Gr. Valley	Colorado R., Colo	18.2	52	85	80	4.4	Coarse grave cobbles
41	Boise	Boise R., Idaho	33	48	48	64	1.9	Compact grave cobbles
42	Sherman Island	Hudson River, N. Y	53	182	161	236	4.4	Sand and boulders
43	Hyatt	Oswegatchie R., N. Y	23	45	37	57	2.6	Sand; glacial boulders
44	Percha	Rio Grande, N. Mex	11	59	73	83	7.6	Sand and gravel
45	Harper	Malheur River, Ore	12	29	57	48	4.0	Sand and grave
46	Moore Weir	Cache Creek, Calif	8	36	30	46	5.8	Sand and gravel
47	Western Jumna Weir	Jumna River, India	8	36	74	61	7.6	Cobbles an boulders

* No failures recorded. † See further description in text.

TABLE 1.—(*Continued*)

Dam No. (1)	Name (2)	Location (3)	Head, in feet (4)	Creep Distance, in Feet: Vertical (5)	Creep Distance, in Feet: Horizontal (6)	Weighted Creep, $\frac{H}{3}$: Distance, in feet (7)	Weighted Creep, $\frac{H}{3}$: Ratio (8)	Foundation material (9)
	*Non-Failures:**							
48	Dam No. 44	Ohio R., Ind.-Ky	9.0	87	35	99	11	Sand, gravel, boulders
49	Granite Reef	Salt River, Ariz	20	45	32	56	2.8	Gravel and boulders
50	Marseilles Weir	Durance R., France	4.3	51	33	62	14.3	Gravel and boulders
51	Weber-Provo	Weber River, Utah	8.5	28	36	40	4.7	Gravel and boulders
52	Strawberry	Span. Fork R., Utah	15	42	63	63	4.2	Gravel and boulders
53	Greeley Intake	Cache La Poudre, Colo	5	34	50	51	10.2	Sand, gravel, boulders
54	Avigononet	France	56	95	171	152	2.7	Gravel, from cobble size to very fine sand
55	Byrmus Paper Company	W. Dudley, Mass	7.6	64	19	70	9.3	Gravel with boulders
56	Radin	Germany	9.7	39	46	54	5.6	Firm gravel
57	Talavera R	Philippine Islands	8.2	18	57	37	4.5	Sand, gravel, boulders
58	Lujan Weir	Argentina	8.2	32	37	45	5.4	Gravel and sand
59	Puntilla Weir	Argentina	15.8	32	50	49	3.1	Gravel and sand
60	Tunuyan Weir	Argentina	8.5	19	31	29	3.5	Gravel and sand
61	Twin City (St. Paul)	Miss. R., Minn	28.5	66	101	99.6	3.5	Sand, gravel, and broken limestone slabs
62	St. Mary's	Sun R. Project, Mont	5.5	20	38	33	6.0	Sand, gravel, boulders
63	Scotland	Shetucket R., Conn	24.4	83	134	128	5.3	Gravel, small boulders
			24.4	82	102	116	4.8	
64	Camp Humphries	Springfield, Va	8	40	12	44	5.5	Coarse sand and gravel
66	Flint R	Flint River, Ga	24	30	105	65	2.7	Broken limestone
67	Oneonta	Susquehanna, N. Y	11	25	30	35	3.2	Sand and gravel
68	Piedmont	West Virginia	12	18	14	23	1.9	Boulders
69	Post Brook	Post Brook, N. J	7.5	50	21	57	7.6	Gravel and boulders
70	Rahway	Raritan R., N. J	7	45	19	51	7.3	Porous gravel
71	Rolling Fork River	Lebanon, Ky	8	30	15	35	4.4	Gravel and boulders
72	Sarda Barrage	Sarda River, India	14	36	138	82	5.9	Shingles and boulders
73	Stamford	Rippowan R., Conn	10	20	16	25	2.5	Gravel and sand
74	Walkill	Front Brook, N. Y	17	14	22	21	1.2	Boulders and gravel
75	Woodstock	Vermont	24	40	40	53	2.2	Loose gravel
76	Juniata River	Huntington, Pa	26	50	60	70	2.7	Cemented gravel overlying hardpan
77	Shoshone	Big Horn R., Wyo	49.5	67	83	94.6	1.9	Boulders
78	Cochiti	Rio Grande, N. Mex	8	17	80	44	5.5	Sand, gravel, cobbles
79	Sixth Lake	Inlet, N. Y	11	18	30.5	28.2	2.6	Gravel and boulders
80	Reservoir	Muskogee, Okla	30	6	105	41	1.4	Gravel and boulders
81	Newton Lower Falls	Boston, Mass	9	40	20	46.7	5.2	Boulders and gravel
82	Dunbarton	Greenwich, Conn	13	36	48	52	4.0	Gravel and boulders
83	Dellwood	Joliet, Ill	20	42	26	50.7	2.5	Sand and gravel
84	Graham Lake	Union River, Me	32	104	90	134	4.2	Gravel and boulders
85	Grants Pass	Rogue River, Ore	38	64	70	87.3	2.3	Gravel
86	Upper Creek	Alameda Cr., Calif	29	70	110	106.6	3.7	Gravel
87	Dam No. 1	Nueces River, Tex	50	120	70	143.3	2.9	Gravel
88	Dam No. 2	Nueces River, Tex	20	62	77	87.7	4.4	Gravel

* No failures recorded.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		(*c*) DAMS ON COARSE SAND						
	Failures:							
1	Flederborn:							
	(Probable)	Kuddow, Germany	23	49	89	79	3.4	Sharp sand underlaid by fin sand
	(Possible)		23	76	89	106	4.6	
	*Non-Failures:**							
11	Upper Coloroon	Coleroon R., India	5.3	24	44	39	7.4	Pure, coarse san
12	Burra	Mahanuddee R., India	12	40	69	63	5.3	Coarse sand
13	Riverdale	Apple River, Wis	24	39	62	60	2.5	Torpedo sand
14	Sidhnai	Ravi R., Punjab, India	7.5	49	53	67	8.9	Coarse sand
15	Godaveri R.	Madras, India	10.8	48	52	65	6.0	Coarse sand
16	Moss Bluff	Oklawaha River, Fla	11.5	159	40	172	15	Graded stratifie sand
17	Florence	Gila River, Ariz	10.5	68	102	102	9.7	Silt and sand†
18	Dam No. 1	Big Sunflower R., Miss	16.9	105.4	34.5	116.9	6.92	Sand with a littl clay, gravel, and clayey sand
20	Mezali	Mon Canal, Burma	11	23	199	89	8.1	Pebbles and san
21	Prairie du Sac							
	Power-house	Wisconsin River, Wis	32	160	141	207	6.5	Pure, coarse san
	Dam		32	130	68	153	4.8	
	Dam-vents open		29	100	24	108	3.7	
22	Columbia	Paulinskill, N. J	26	66	31	76	2.9	Coarse sand
			26	64	35	76	2.9	
23	Cranford	Rahway River, N. J	6	16	20	23	3.8	Coarse sand
24	Gloucester	Massachusetts	23.5	42	35	54	2.3	Coarse sand
			19.5	42	68	65	3.3	
25	C. Humphries	Accotink Cr., Va	8	40	12	44	5.5	Coarse sand to gravel
26	Bound Brook	Raritan R. and Millstone R., N. J	5.7	6	24	14	2.4	Coarse sand
27	Upper Nile Canal	Colorado	16	60	31	70.3	4.4	Coarse sand
28	Watertown	Wisconsin	15	48	20	54.7	3.65	Coarse sand
29	Leavittsburg	Ohio	15	42	58	61.3	4.09	Coarse sand
30	Highland Pk	Detroit, Mich	14.5	4	140	50.7	3.5	Coarse sand
31	Gillespie	Gila River, Ariz	20	60	68	82.7	4.1	Sand and gravel
32	Garfield	S. Pasadena, Calif	16	6	80	32.7	2.05	Sand and gravel
33	Wausau	Wisconsin R., Wis	32	230	98	262.6	8.20	Coarse sand
		(*d*) DAMS ON SAND, FINE SAND, AND SILT						
	Failures:							
1	Narora	Ganges, India	13	45	50	62	4.7	Fine sand
				44	99	77	5.9	
				20	50	37	2.8	
2	Khanki (Lower Chenab)	Punjab, India	12	15	104	50	4.2	Fine sand
3	Corpus Christi	Nueces, Texas	37	39	115	77	2.1	Sand
4	Deoha	Deoha, India	12	60	180	120	10.0	Fine sand
8	Kitcha Bye	Rohilkhand, India	8	22	36	34	4.3	Dry sand
9	Kulli Dam	Rohilkhand, India	......	......	......	......	......	Fine sand
10	Kulli Bye	Rohilkhand, India	......	......	......	......	......	Fine sand
11	Nadrai Escape Fall	Lower Ganges, India	20	33	76	58	2.9	Light sandy soil
12	Harmon Park	Lebanon, Ohio	6	6	9.6	9.0	1.5	Quicksand and gravel
			6.5	8.5	11	12	1.9	
	Non-Failures: *							
18	Deoha	Deoha R., India	12	140	180	200	16.7	Sand
19	Kabo Weir	Mu River, Burma	10.8	42	177	101	9.4	Sand
			10.0	58	129	101	9.4	
20	Delhi	Sone, India	8.0	26	112	63	7.9	Clay and sand
21	Dowlarshwaram	Godaveri Delta, India	10	26	164	81	8.1	Sand
22	Ferozepore Weir	Sutlej, India	22	92	192	155	7.0	Fine sand at surface; then coarse sand to clay
23	Islam Weir	Sutlej, Punjab	18	79	175	137	7.6	Sand
24	Jamrao	Sind, India	8	42	111	79	9.9	Fine sand

* No failures recorded.

† See further description in text.

TABLE 1.—(*Continued*)

Dam No.	Name	Location	Head, in feet	Creep Distance, in Feet		Weighted Creep, $\frac{H}{3}$		Foundation material
				Vertical	Horizontal	Distance, in feet	Ratio	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	*Non-Failures:**							
25	Khanki	Punjab, India	13	46	168	102	7.8	Fine sand
			13	75	135	120	9.2	Fine sand
26	Adimapalli	Penner, India	8.5	54	68	77	9.0	Sand and rock
27	Lloyd	India	18.5	114	190	177	9.6	Sand and silt
28	L. Jhelum	Jhelum River, India	10	57	195	122	12.2	Fine sand
			10	92	134	137	13.7	Fine sand
29	Merala	Chenab River, India	12.5	131	121	171	13.7	Fine sand
30	Narora	Ganges R., India	13	59	142	106	8.2	Fine sand
31	Nadrai Escape Fall	Lower Ganges, India	20	19	176	78	3.9	Light sandy soil
32	Nzeswaran	Godaveri Delta, India	10.7	46	130	89	8.3	Sand
33	Panjnad Weir	India	9	92	202	159	17.7	Sand †
	Panjnad Annex Weir	India	10	139	163	193	19.3	
34	Assuit	Nile River, Egypt	8.2	53	138	99	12	Water-laden fine sand
35	Isna (Esna)	Nile River, Egypt	8.2	52	139	98	12	Fine sand
36	Nag Hannadi	Nile River, Egypt	14.7	72	195	137	9.3	Fine sand; some coarse
37	Zifta	Nile River, Egypt	13	54	156	106	8.1	Clay, sand, and mud †
38	Hindia	Euphrates, Mesopotamia	16.4	69	292	166	10	Fairly hard silt
39	Sultan Bend	Murgab R., Turkestan	24	168	207	238	10	Probably loess
40	Su Chuang:							
	Regulator	Pei Ho, China	11.5	83	91	113	9.9	....
	Sluice		13.1	83	116	121	9.3	Loess and sand
41	Versen	Ems River, Germany	8.6	76	28	85	10	Sand
42	Ketchetovskaia	Don River, Russia	11.2	90	116	129	11.5	Sand
43	Angat River	Angat, Philippine Islands	9.8	83.4	53.2	101	10.2	Sand, gravel; some mud
44	Dam No. 48	Ohio R., Ind.-Ky	9	84	35	96	10.6	Very fine sand
45	Dam No. 51	Ohio R., Ill.-Ky	8	87	35	99	12.3	Fine sand; some gravel
46	Dam No. 47	Ohio R., Ind.-Ky	9	87	35	99	11.0	Fine to medium sand
47	Rockwood Heading	Colorado R., Calif	4.5	56	45	71	15.8	Very fine silt
48	Middle Loup	Mid. Loup R., Nebr	10	134	13	138	13.8	Sand, probably fine
49	Iron Mt	Menominee R., Mich	30	119	138	165	5.5	Fine sand
50	Mendota	San Joaquin R., Calif	10	56	50	72.6	7.3	Drifted sand
51	Cambian	Nepan River, New South Wales	6	28	62	49	8.2	Sand
52	Sandy Lake	Minnesota	12	129	115	167	14.0	Sand
53	Bonnet Carre	Mississippi R., La	14.5	149	103	183	12.6	Loam, silt, and clay
54	Beardsley Canal Weir	Kern River, Calif	7	32	84	60	8.6	Fine sand and silt
55	Leasburg	Rio Grande, N. M. and Tex.	7	50	30	60	8.6	Sand
56	Mottville							
	Plant:							
	Sec. BB	St. Joseph, Mich	13	38	66	60	4.6	Quicksand
	Sec. CC		13	52	41	66	5.0	
	Sec. DD		13	48	40.3	61.4	4.7	
			17.5	70	28	79	4.5	Sand
57	Built for	Lowell, Mich	17.5	52	2	53	3.0	
			15	54	44	69	4.6	Fine, clear sand
58	Built for	Bellaire, Mich	15	36	30	76	3.1	
59	Akron	Cuyahoga, Ohio	35	115	100	148	4.2	Sand and silt
60	Anadarko	Washita, Okla	26.5	65	57	84	3.2	Sand
			18	40	30	50	2.8	Sand, fine sand, silt
61	Barre	Massachusetts	15	50	67	72	4.8	
62	Miami	Miami R., Ohio	10	28	39	41	4.1	Sandy silt
63	Oklahoma	Canadian R., Okla	8	40	30	50	6.3	Sand and shale
64	Wiggins	Nice Canal, Colo	10	48	41	62	6.2	Quicksand
65	Okla. City	N. Canadian R., Okla	12.0	38	34	49.3	4.1	Fine sand and silt
66	Cotulla Dam	Nueces River, Tex	22	60	75	85	3.9	Fine sand and silt
67	Rogers Dam	Muskegon R., Mich	35	90	89	119.6	3.4	Sand and silt
68	Oradell Dam	Hackensack R., N. J	16	102	72	126	7.9	Sand and silt
69	Milliken	Nueces River, Tex	22	52	82	79.3	3.6	Fine sand and silt
70	Croton Dam	Muskegon R., Mich	39	128	275	219.6	5.6	Sand and silt
71	Wichita River	Wichita Falls, Tex	9.5	32	180	92	9.7	Sand
72	Rice's Rip	Maine	28	96	66	118	4.2	Sand

* No failures recorded.

† See further description in text.

The second fact is based on the data on dams given in Table 2, which show that much lower ratios than were recommended by Bligh or Griffith have been used successfully. As Bligh and Griffith do not give ratios for material of the exact description of that existing at some of these dams, the writer has used his best judgment in estimating from the classifications which they did give, what they would have probably recommended.

TABLE 2.—COMPARISON OF CREEP-HEAD RATIOS

Dam	Material	Creep-head ratio	RECOMMENDED RATIOS (APPROXIMATE)		ACTUAL RATIO IN PERCENTAGE OF RECOMMENDED:	
			Bligh	Griffith *	Bligh	Griffith
Dam No. 48, Ohio River..	Very fine sand........	13.2	18	11.6–12.8	73	103–113
Dam No. 47, Ohio River..	Sand, very fine to medium; a little fine gravel	13.5	18	11.6–12.8	75	105–116
Barre, Mass.............	Fine sand and silt.....	8.0	18	11.6–12.8	44	62– 69
Iron Mountain...........	Fine sand..............	8.6	15	10 –11.2	57	77– 86
Riverdale...............	Coarse sand..........	4.2	12	8 – 9.6	35	44– 52
Prairie du Sac..........	Coarse sand..........	4.3	12	8 – 9.6	36	45– 54
		6.2	12	8 – 9.6	52	65– 77
Columbia...............	Coarse sand..........	4.0	12	8 – 9.6	33	41– 50
Dam No. 1, Big Sunflower River................	Sand with a little clay, gravel, and clayey sand	6.6	14	8	47	83
Tunuyan Weir...........	Sand and gravel.......	6.0	9	6.4	66	94
Granite Reef............	Gravel and boulders...	3.2	5–9	4.8	36–64	66
Walkill, N. Y............	Gravel and boulders...	2.6	5–9	4.8	30–52	54
Oswegatchie River.......	Sand and boulders....	3.6	5–9	6.4	40–72	54
Piedmont...............	Boulders.............	2.7	4	3.2	68	85

* These values include the reduction for vertical staunching.

Considering the two failures previously mentioned, in which the creep distances were substantially equal to the recommended values in connection with the ratios in Table 2 (in which the ratios are much less than recommended), it is a significant fact that the group that failed had nearly all the creep distance horizontal and the other had substantial parts of its percolation distances along vertical or steeply sloping surfaces. In fact, not a single case was found in which piping unquestionably took place under a deep cut-off, and only one case, the Flederborn Dam (Table 1(*c*) Dam No. 1), in which it may have. The data on this failure are so incomplete that the piping may have been behind the abutments, rather than under the dam; but even if it was under the dam it probably took place at ratios much less than those recommended in this paper and, certainly, at ratios somewhat less. The failures of the Coon Rapids and Plattsburgh Dams (Table 1(*b*), Dams Nos. 4 and 5) were apparently due to defective construction, and that of the Pittsfield Dam (Table 1(*b*), Dam No. 3), to a blow-out directly through the foundation material from a porous layer beneath the impervious layer on which the dam was founded and not along the line of creep.

The logical conclusion from these facts is that in the design of masonry dams on earth foundations, greater weight should be given to the creep along vertical or steeply sloping surfaces than along horizontal or slightly sloping ones. This has led the writer to the development of the weighted-creep analysis of existing dams and dam failures.

ORIGIN OF THE WEIGHTED-CREEP ANALYSIS

The writer's attention was first directed to the greater value of vertical creep by the difficulty of explaining the stability of the Prairie du Sac Dam, in Wisconsin (Table 1(*c*), Dam No. 21), with its extremely low plain creep ratio of 4.3 on a coarse sand foundation. The evidence from one structure, however, was not sufficient to justify a general conclusion.

Frequently, when one forms a correct impression and thoroughly studies the literature of the subject he finds the same idea expressed by others. There are few absolutely unique ideas in engineering. The idea apparently was expressed first by Griffith, who suggested in his abridged paper[6] that a reduction could be made in his suggested creep-head values for reliable vertical staunching. He explains the greater effectiveness of vertical creep as follows:

> "First, the line of creep is subject to a greater pressure where carried to a greater depth; secondly, at the greater depth larger and heavier sand will probably be met; and, thirdly, sand carried out must be lifted up the curtain wall, presumably requiring greater velocity of percolation."

Mr. Griffith kindly supplied the data on the Kulli Bye-Wash and expressed the conviction that his creep-head ratios were probably too low unless some vertical staunching was used. The writer's explanation for the difference seems to have been independently reached by Mr. A. A. Musto[22].

The effect of sheet-piling in reducing upward pressure and cutting down the flow through the voids in the foundation material has been discussed at length by Khosla[15] and Leliavsky[14]. Both seem to have reached the conclusion that such piling reduces the pressure by a greater amount per unit of contact length than horizontal creep.

RELATIVE WEIGHTS OF VERTICAL AND HORIZONTAL CREEP

The proper relative weight to give to vertical and horizontal creep can be determined only by the analysis of actual dams, although some light may be thrown on it by an analysis of observation of upward pressure on actual dams. While the results of the analyses of all available data on percolation distances of dams do show conclusively that the horizontal creep distance is not as effective in resisting piping as vertical creep distance, they do not indicate exactly what their relative effectivenesses are. Although several dams without appreciable vertical creep have failed with distances that normally would be considered sound, no failures were found with considerable vertical creep, except at ratios much less than those ordinarily considered safe, and, therefore, the upper limit of weight must remain uncertain.

A careful study of the upward pressure measurements on several structures on which data were available shows that the drop in upward pressure along horizontal concrete surfaces is almost zero, and along a puddle surface somewhat more, indicating that for horizontal concrete surfaces the ratio might be as low as zero. It does not necessarily follow, however, that the safe ratio for horizontal creep in considering piping failure is the same as in considering upward pressure. More study along this line is very desirable.

---

[22] *Paper No. 142,* Punjab Eng. Congress, 1930, p. 212 *ƒ*.

In arriving at the decision to use a weight of one-third for horizontal creep the writer listed all the structures having creep distances less than, or near, what might be considered the lowest safe limit, together with the material on which they were founded and the weighted creeps for each, with weights for horizontal creep of 0.5 and 0.33. At the same time, a table was made of the weighted-creep ratios which the results of analyses of actual dams indicated to be safe for the various creep ratios.

In view of the uncertainty as to how low a ratio to use for horizontal creep, it is advisable to be conservative. As the present use of unity for this weight has generally produced safe structures (although frequently unnecessarily conservative), it is not desirable to depart too far from the present standards until data definitely establishing the safety of the new ones are available. The writer, therefore, recommends a weight of one-third for the horizontal creep, although there are considerable data to indicate that a higher ratio might be better.

It should be recognized that the one-third weight may not be the best possible ratio in all cases. Further research may show that it should be varied, depending on a number of factors, and study to bring about such relations should be encouraged. The ratio recommended, however, seems to be the best that can be adopted with the present knowledge. To recommend a definite quantity involves the danger that it will be followed blindly by some, without consideration being given to the manner in which it was derived. There seems to be no way by which this danger can be avoided. Since the studies have shown that, for the present, the one-third ratio seems to be the best that can be reached, to fail to give it would require that each individual dam designer go through a long process of deriving a value which he considered satisfactory, and unless he had many new data, it is unlikely that he would arrive at one materially better than that suggested. It is doubtful whether many designers, particularly when working on small dams, would have the time available for such studies. The best method, therefore, seemed to be to give the value which from the available information, seemed to be most satisfactory, with the warning that it probably did not represent a final solution, and to present, as fully as possible, the data on which it was based, in order that those who desired might arrive at their own conclusions, and that when further data became available a new determination might be made. If no value of weighted creep was suggested, those who might adopt it blindly would equally blindly adopt one of the old Bligh values; and since this cannot be prevented, it would seem better to have them adopt a weight of one-third, since it would probably give them a better creep distance.

SUGGESTED WEIGHTED-CREEP RATIOS

In Table 3 are given the weighted-creep ratios which the analysis of all available data from existing structures (and particularly from those in Tables 1 and 2) indicate as necessary for safety against failure from piping along the contact of the structure and its foundation. In order to use these values with safety the cut-offs must be of solid masonry built in contact with the earth sides of the trench, or of interlocking steel or concrete piling driven so that the interlock is not broken, and so that it is satisfactorily embedded at the top in the masonry structure. They also assume competent supervision during construction and efficient maintenance afterward. These values are for major structures. Somewhat smaller values may be used for less important structures, ranging down to perhap 80% of those given for those of minor importance.

TABLE 3.—Comparison of Weighted-Creep Ratios
(Weight of Horizontal Creep, One-Third)

Material	Safe weighted-creep ratio	Bligh's value
Very fine sand or silt	8.5	18
Fine sand	7.0	15
Medium sand	6.0	...
Coarse sand	5.0	12
Fine gravel	4.0	...
Medium gravel	3.5	...
Gravel and sand	...	9
Coarse gravel, including cobbles	3.0	...
Boulders with some cobbles and gravel	2.5	...
Boulders, gravel, and sand	...	4 to 6
Soft clay	3.0	...
Medium clay	2.0	...
Hard clay	1.8	...
Very hard clay, or hardpan	1.6	...

There are so many types of foundation material that it is impossible to give values for all. Only the usual types are given, therefore, and the other conditions can be determined by comparison with these. The values for medium and soft clay are somewhat uncertain as no record was found of dams founded on these materials. If the requirements of bearing pressures can be met, the values given would seem to be sufficiently conservative.

Recommended Values Are Conservative

The recommended values have intentionally been made quite conservative. Not a single failure was found where the dam had creep-head ratios as large as those given. It is possible that future experience may show that lower values are unquestionably safe. Quite a number of dams with lower values have been built and have given satisfactory service over a period of years. Some experienced engineers use considerably lower values. In a number of cases, however, failures have occurred suddenly and without previous warning in structures that have given satisfactory service over a long period, ranging up to twenty years. If a set of values which were somewhat too low came into general use, it might be a long time before their insufficiency would be demonstrated and a large number of insecure structures would be built in the meantime. It is better, therefore, to take a conservative view and make a small step in the right direction permitting more experience to accumulate before taking another step, than to run the risk of overstepping and having to go back.

The present form of this paper has been a gradual development and is the third step in this process which the writer has made. In the first report which was based on data from approximately 100 dams, he recommended the plain creep theory with somewhat lower values than those recommended by Bligh. As data from the Deoha Barrage and Kulli Bye-Wash became available, the weakness of the plain creep analysis became evident and a second report was prepared, based on data from 120 dams recommending a set of values with a weight for horizontal creep of one-half. Later data, largely furnished by the firms of Ambursen Dam Company; Ayres, Lewis, Norris, and May; and Holland, Ackerman, and Holland which included many dams with a high proportion of vertical creep, gave adequate basis for using smaller relative weight to horizontal creep. The present paper was prepared, therefore, based on a study of more than 200 dams, using a weight for horizontal creep of one-third.

Low values of creep for dams founded on sand have been recommended by Koenig.[8] The shape of dam and sheet-piling advocated by him give weighted creeps varying from 8.5 for low dams to 3.6, or less, for high ones. It is doubtful, however, whether his recommendations for higher heads were based on actual experience.

An examination of minimum weighted-creep ratios suggested as a result of this study shows that they are somewhat less than one-half those recommended by Bligh. Table 3 offers a comparison.

This does not mean that Bligh's values are more than 100% too large, or that dams designed according to the weighted-creep ratios will have only one-half the percolation distance that dams have had in the past. The difference does not represent as radical a change from the present custom as a comparison of the ratios would suggest. The requirements of length to protect against scour, and to secure sufficient bearing pressure, will still necessitate considerable horizontal creep. For dams, on coarse gravel, including cobbles and boulders with some cobbles and gravel, the necessity of conserving all the water in some cases may require greater percolation distances, because the values given may permit too much seepage.

The effect of the adoption of the weighted-creep method will be to put the emphasis on deep cut-offs, and will lead to their use as far as other limitations will permit. It will tend to eliminate the wide masonry-floor type without substantial cut-offs. For example to meet the weighted-creep requirements for a weir on fine sand with no vertical creep, would require an ordinary creep distance of 3 × 7 = 21 times the head, while the ordinary creep distance according to Bligh should be 15 and according to Griffith, 12½ to 16.

Where it is possible to obtain a detailed knowledge of the extent and position of impervious or relatively impervious layers that may exist, the dam should be designed in the light of this knowledge in which case the recommended values would be merely guides. They are intended for use directly only where such information is not available, or where the borings indicate no persistent or well-defined impervious parts of the foundation material.

Because of the emphasis which the weighted-creep method puts on deep cut-offs if the greatest economy is to be secured, it will be necessary to investigate more fully than before the length of apron necessary to prevent scour, and the best shape of dam to accomplish this purpose. The pressure on the designer will be toward using as short an apron as possible and unless careful studies are made there will be a tendency to adopt one that is too short. It should be kept in mind that the weighted-creep method deals with piping only, and the requirements of scour may dictate much more expensive structures than safety from piping would necessitate. The recent advances in hydraulic model tests have clearly shown the reliability of this means of determining the conditions necessary to prevent scour. The design of important structures, therefore, should include a hydraulic model study.

Because of this emphasis on deep cut-offs there will be a tendency to use deep, single lines of sheet-piling. Evidence is accumulating, however, which shows that it is unwise to "put all one's eggs in one basket," and for important structures especially two lines of shorter piling would be much preferable to one line of long piling, as there would then be a "second line of defense". Failures of these structures have already been recorded due to defects in the sheet-piling. These sometimes occur even with reliable engineers and contractors, and the probability of serious results is much less with two lines of piling.

Grouting Beneath Horizontal Parts of Dams

Since the available evidence indicates that there is likely to be a poor contact between the foundation and horizontal parts of dam foundations, improving this contact by grouting is worthy of consideration. This could usually be done without great expense through holes drilled in the floor-slabs. High pressures could not be used because they would lift the floor By starting at one end of the structure and working toward the other, it is believed that good contact could be secured. If possible, the grouting should be done before the head came on the structure, as otherwise much of the cement might be carried away in the flowing water.

Reverse Filters, Weep-Holes, and Drains

Another factor that should be considered in selecting the proper ratio to use in the design of a masonry dam on a porous foundation is the possibility of using reverse filters, weep-holes, or drains. The reverse filter was probably first developed by Mr. J. S. Beresford and seems to have been used extensively about 1914. It was used on the Zifta and Assuit Barrages, on the Nile River, and on the Hindia Barrage, in Mesopotamia (Table 1(*d*), Dams Nos. 34, 37, and 38). It has been used in this country on Dam No. 7, at Amsterdam, N. Y., and Dam No. 13, at Yost, N. Y., on the Mohawk River (Table 1(*b*), Dams Nos. 35 and 36). It consists of a filter placed at the lower edge of the down-stream apron, built up of a bottom layer of fine material surmounted by other layers of progressively coarser material. It is called a reverse filter because the position of the layers is the reverse of that in ordinary filters, but the water passes first through the fine and then through the coarse material as in other filters. Its only function is to filter the water seeping beneath the dam and to prevent the removal of any of the fine foundation material. Although usually covered with rip-rap, the reverse filter is likely to be damaged or destroyed by the water passing over the dam at high velocity.

Weep-holes through the apron of the dam or the wing-walls have been extensively used in America, but no examples have been found of their use in other countries. Most of the available literature on India and Egypt, however, was published many years ago, and it is possible that they have come into use there since then. Their function is usually to relieve upward pressure beneath the apron of the dam and thus to permit the use of a thinner apron without danger of its being lifted from under-pressure. In recent years, pipe drains have been utilized in some cases instead of weep-holes. The greatest advantage is secured by combining the functions of the reverse filter and weep-hole by constructing a filter behind the weep-holes or around the pipe drains. In this position the filter is protected from scour.

Since piping failure results from the removal of foundation material from beneath the dam, if a filter prevents the removal of this material, the dam will be safe, even if large quantities of water are passing beneath it. The quantity that passes through the filter depends on the head acting on it. In ordinary filtration practice, a filter is frequently subjected to a head of water several times its thickness. If an ordinary reverse filter were subjected to such a head it would be lifted bodily. Such filters, therefore, cannot be subject to much head. However, if the filter is weighted down by a sufficient thickness of concrete apron, it can be subjected to a much higher head than a reverse filter, and the dam will thus be made safe for the passage of a greater quantity of water beneath it. With such a filter near the end of the percolation path of the water flowing beneath a dam, a greater velocity of flow in this water is permissible and, therefore, a shorter percolation path can safely be used.

Weep-holes and drains have been used with very short percolation distances between them and the head-water. In one case, a flow through the weep-holes sufficient to cause geysers several inches high occurred without apparent ill effects.[23]

A striking example of the effectiveness of vents came to light at the Coon Rapids Dam, in Minnesota (Table 1(*b*), Dam No. 4). This is an ogee structure having a thin down-stream apron with steel sheet-piling cut-offs under the heel of the dam at the down-stream edge of the thin apron. Immediately behind the up-stream cut-off a drain was constructed, consisting of a reverse filter discharging into a small tunnel built in the dam. Due to an accident, one of the steel sheet-piles of the up-stream cut-off was pushed down far beyond its proper position, leaving an opening in the cut-off, and reducing the creep distance to the drain to 0.6 times the head. In spite of his the flow through the drain does not seem to have been sufficient to excite larm and the defect in the cut-off was not discovered until a large hole vashed below the dam was pumped out during repairs to the apron and a low-out occurred. Had the drain not existed it seems quite probable that he thin concrete apron below the dam would have been blown up.

---

[23] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 319.

Table 4 gives data on dams using weep-holes, drains, or reverse filters, ogether with the creep head, weighted-creep head, and short-path ratios to the most up-stream vent, and other pertinent data. The low ratio resulting from the short distance from the head-water to the up-stream vent in many cases is striking.

The writer does not believe that it is safe to use low ratios for drains or weep-holes, except where they are combined with a reverse filter. It is probable that the use of a drain without a filter contributed to the failure of the abutment of the Corpus Christi Dam (Table 1(*d*), Dam No. 3). For many years, the U. S. Bureau of Reclamation has used weep-holes with filters of gravel beneath with good success. Experience at the Lost River Dam[24] shows that weep-holes without filters, or other coarse material, behind them are likely to clog. Although weep-holes and drains are valuable devices, the greatest care must be exercised in their construction. One that will permit the foundation material beneath the dam to pass through in effect, is a pipe already partly formed, and certainly no such weak spot in the protection of the dam should be constructed deliberately with the impression that it was adding to the safety of the structure.

Weep-holes or drains have been used on dams of practically all classes of material. The necessity of a well-constructed filter is especially evident in the case of weep-holes or drains with clay, silt, or fine sand foundations, as the fine material could readily pass out through them unless restrained by an effective filter. In gravel, a filter probably would form naturally by the washing out of the fine material near the weep-hole or drain, in the same manner as a filter tends to form around the screen of a driven well. However, in view of the small expense in installing a filter behind the weep-hole, it is believed that their construction is justified even in gravel. They are absolutely necessary in clay or silt, if weep-holes are depended upon to reduce the length of the percolation path required.

The location of the weep-holes and filters should be given considerably study in dam design. From the standpoint of the prevention of piping, the best location for the filter is probably beneath the apron at the downstream end just up stream from the cut-off wall or row of sheet-piling which is usually placed at the down-stream edge of the structure, the weep-holes discharging through the wall or piling into the rip-rap or other scour protection material. The down-stream wall or row of piling in this case does not act so effectively as a cut-off, but is very desirable to prevent undermining. It need be no longer, therefore, than is necessary to accomplish this purpose. To reduce the upward pressure on the apron it may be necessary to place the weep-holes farther up stream. They should not be placed

---

[24] *Engineering News*, Vol. 71, April 30, 1914, p. 968.

TABLE 4.—DATA ON WEEP-HOLES, DRAINS, AND REVERSE FILTERS (FOR DESCRIPTIVE NOTES, SEE TABLE 1)

Dam	Type	Head	PLAIN METHOD				WEIGHTED METHOD, $\frac{H}{3}$				SHORT-PATH METHOD				Remarks
			Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	Total distance	Creep ratio	Total distance to weep-holes	Vent ratio	
						(a) WEEP-HOLES									
Aganca R.; Irr. Sys......	Ogee, crest and apron..	9.4	106.9	11.4	86.2	9.2	81	8.6	60	6.4	74	7.9	63.1	6.8	Weep-holes
Santa Barbara; Irr. Sys...	Ogee, crest and apron..	9.9	101.5	10.3	94.7	9.6	70	7.0	70	7.0	70.9	7.2	67.9	6.9	Weep-holes
Bassano, Canada.........	Amburseu............	46	199	4.3	36	0.8	89	1.5	27	0.6	177.5	3.9	29	0.6	....
Vandalia................	Ambursen............	27	304	11.3	151	5.6	177	6.5	99	3.7	....	....	....	....	....
Upper Grays............	Ambursen............	30	155	5.2	30	1.0	82	2.7	27	0.9	....	....	....	....	....
Batavia, N. Y...........	Ambursen............	10	51	5.1	14	1.4	28	2.8	13	1.3	41.6	4.2	12.3	1.2	....
Bethlehem, N. H.........	Ambursen............	23	156	6.8	66	2.9	119	5.2	62	2.7	....	....	66	2.9	....
Burnham, Me............	Ambursen............	32	173	5.4	35	1.1	103	3.2	31.7	1.0	....	....	35	1.1	....
Kearsley Creek, Mich.....	Ambursen............	25	244	9.8	67	2.7	152	6.1	66.3	2.7	....	....	67	2.7	....
Mayfield, N. Y..........	Ambursen............	30	185	5.1	45	1.5	85	2.8	41.7	1.4	....	....	45	1.5	....
Massillon, Ohio..........	Ambursen............	25.5	102	4.0	32	1.3	63	2.5	30	1.2	....	....	32	1.3	....
Newport, N. H..........	Ambursen............	16.5	98	5.9	35	2.1	61	3.7	34.3	2.1	....	....	35	2.1	....
Prince Albert, Can.......	Ambursen............	23	175	7.6	28	1.2	92	4.0	23	1.0	138.2	6.0	23	1.0	....
Wakeman, Ohio..........	Ambursen............	15	72	5.1	18	1.2	44	2.9	16.7	1.1	57	3.8	16	1.1	....
Hightstown, N. J........	Ambursen............	11	58	5.3	29	2.6	39	3.6	27	2.5	....	....	29	2.6	....
Wonder Lake, Ill.........	Ambursen............	20	93	4.7	21	1.1	48	2.4	20.3	1.0	....	....	21	1.1	....
Pittsfield...............	Ambursen............	33	76	2.3	24	0.7	45	1.4	20	0.6	63	1.9	21.5	0.7	....
Miller..................	Concrete stop-logs.....	5	38	7.6	28	5.6	28	5.6	19	3.9	25	5.0	20	4.0	....
Fremont................	Ogee, crest and apron..	3.5	56	16.0	48	13.7	32	9.1	28	8.7	43	12.3	39	11.1	....
Horse Creek............	....	6.4	36	5.6	29.7	4.6	26	4	21	3.3	29	4.5	25.7	4.2	....
Mesilla.................	Ogee, crest gates, apron	6.7	67	7.7	57	6.6	39	4.5	30	3.5	51	5.9	45	5.2	....
Grand Valley............	Ogee, with roller crest..	18.3	137	7.5	117	6.4	80	4.4	63	3.5	100	5.5	92	5.0	....
Oswegatchie............	Ambursen............	23	82	3.6	37	1.6	57	2.5	22	1.0	57	2.5	30	1.4	....
Hyatt, N. Y............	Ambursen............	26	86	3.3	37	1.4	57	2.2	33.6	1.3	60	2.3	32.2	1.2	....
Percha.................	Ogee, with apron......	11	132	12	100	9.1	83	7.6	66	6.0	86	7.8	64	5.8	....
Harper Diversion Dam...	Gate with apron.......	12	86	7.2	78	6.5	48	4	40	3.3	69	5.8	63	5.3	....
Granite Reef...........	Ogee, with apron......	20	77	3.8	58	2.9	56	2.8	41	2.0	58	2.9	47	2.4	....
Weber-Provo...........	Ogee, with apron......	8.6	64	7.5	52	6.1	40	4.7	29	3.4	50	5.9	45.	5.3	....
Strawberry.............	Ogee, with apron......	15	105	7.0	59	3.9	63	4.2	42	2.8	84	5.6	43	2.9	....
Burmis Paper Co. Dam...	Ogee, with apron......	7.6	83	10.9	74	9.7	70	9.2	62	8.1	66	8.7	59	7.8	....
Talavera River; Irr. Sys...	Ogee, crest and apron..	8.2	75	9.2	71.0	8.7	37	4.5	33	4.0	68.5	8.4	66.5	8.1	Section at east half of dam
		7.2	99	13.7	96.2	13.3	45.8	6.3	42.8	5.9	86.0	11.9	84.3	11.7	Section at west half of dam
Twin City Dam..........	Ambursen............	30.5	197	6.5	134	4.4	104	3.4	80	2.6	165	5.4	107	3.5	Weep-holes
St. Mary's Intake Dam...	Ambursen............	5.5	57	10.4	48	8.6	33	6	83	4.2	43	7.8	39	7.1	....
Uncas (Scotland Dam)....	Ambursen............	24.4	184	7.5	33	1.4	128	5.3	33	1.4	114	4.7	33	1.4	....
Rahway, N. J...........	Ambursen............	7.3	64	9.1	23.5	3.4	51	7.3	23	3.3	40.5	5.8	23	3.3	

(a) WEEP-HOLES—(*Continued*)

Wallkill	Ambursen	17	35.5	2.1	10.5	0.6	21	1.3	8.8	0.5	36	-2.1	9.0	0.5	....
Juniata	Ambursen	26	110	4.2	30	1.2	70	2.7	30	1.2	81.5	3.2	30	1.2	....
Sixth Lake Dam	Ambursen	11	48.5	14.4	18.5	1.7	28.2	2.5	15.5	1.4	40	3.6	18.5	1.7	....
Florence	Ogee, with apron	10.5	170.0	16.2	143	13.6	102	9.7	93	8.9	112	10.7	85	8.1	....

(b) DRAINS

French Landing Power Plant	Multiple-arch hollow dam	32	104	3.2	58	1.8	68	2.1	53	1.7	100	3.3	50	1.6	Drains installed
Shawano	Ogee, crest gates, apron.	15	138	9.2	76	5.1	84	5.6	43	2.9	....	....	....	....	Gravel drain
Barton Power Plant	Multiple-arch dam	25	93	3.7	48	1.9	58	2.3	43	1.6	85	3.4	41.5	1.7	Drains installed
Black River Storage Dam	Multiple-arch dam	15	110	7.4	35	2.3	75	5.0	33	2.2	97.5	6.5	34	2.3	Drains installed
Des Moines Elec. Co., Ia.	Hollow multiple-arch dam	8	55	6.8	22	2.8	33	4.1	19.3	2.4	58	7.3	22	2.8	Drains used
City of Sturgis, Michigan	Hollow multiple-arch dam	25	....	....	52	2.1	....	....	45	1.8	....	....	43.9	1.8	Drains installed
		25	....	....	20	0.8	....	....	15	0.6	....	....	20	0.8	....
Coon Rapids	Ogee, crest gates, apron.	48	131	2.7	....	....	45.8	9.5	....	....	130	2.7	....	....	With filter
		20	....	....	12	0.6	....	....	9	0.5	....	....	12	0.6	With filter
Iron Mountain	Ogee, crest gates, apron.	30	257	8.6	171	5.7	165	5.5	122	4.0	180	6.0	129	4.3	....
Village of Lowell, Mich.	Ambursen	17.5	100	5.6	54	3.1	81	4.6	53	3.0	86	4.9	50	2.9	Spillway section Drains installed
Village of Bellaire, Mich.	Ogee, crest gates, apron	15	98	6.6	30	2.0	69	4.6	31	2.1	83.4	5.6	30	2.0	Section through spillway
		15	66	4.4	32	2.1	46	3.1	31	2.1	64.4	4.3	32	2.1	Section through power house Drains installed

(c) REVERSE FILTERS

Sandorab	Ogee, with apron	16.7	165	9.9	....	....	93	5.6	....	....	113	6.8	....	....	Filter at end of dam
Dam No. 7, Mohawk (Amsterdam)	Bridge	12	95	7.9	....	....	53	4.4	....	....	67	5.6	....	....	Filter at end of dam
Dam No.31, Mohawk (Yost)	Bridge	8	76	9.5	....	....	42	5.3	....	....	59	7.4	....	....	Filter at end of dam
Hindia Barrage	Stoney gate	11.5	....	....	115	10.0	....	....	52.5	4.6	....	....	102	8.9	....
		16.4	361	22.0	....	....	164	10	....	....	309	18.9	....	....	....
Panjhad Annex Weir	Flat weir	10	363.0	36.3	305	30.5	235	23.5	19.6	19.6	222	22.2	199	19.9	....
Assuit Barrage	Stoney gate	8.2	191	23.3	....	....	98.5	12	....	....	148	18	....	....	Filter at end of dam
Zifta Barrage	Stoney gate	13	211	16	....	....	106	8.2	....	....	160	12.3	....	....	Filter at end of dam
Leasburg Dam	Ogee, crest gates, apron	7	80.7	11.4	70	10.0	60	8.6	50	7.1	58.5	8.4	156	8	Sheet-piling was not tight

farther than necessary for this purpose (unless required by safety from sliding) because to do so shortens the percolation path with no advantage, but brings a greater pressure on the filter and, therefore, increases the danger of failure. For this reason the installation of a drain immediately down stream from the up-stream cut-off is not good practice. On sand or silt foundations where flexible block or book-slab pavements are used down stream from the solid apron of the dam, it may be desirable to install weep-holes and a filter at the down-stream edge of the apron. With the block protection, the water passing beneath the dam is prevented from escaping from a large part of the area just down stream from the apron, but is free to escape through the cracks between the blocks. This tends to produce concentrated flows through these cracks and causes the removal of fine material, making an incipient pipe.

A word of caution also may not be amiss regarding drains beneath the apron of dams where a hydraulic jump is formed. If the drain discharges down stream from the jump while its position under the apron is up stream from the jump, the pressure beneath the apron will be equal to, or greater than, that where the drain discharges, which, being below the jump, will be greater than the pressure acting downward on the apron, due to the thin sheet of water up stream from the jump. This unbalanced pressure may be enough to lift the apron. In the Iron Mountain Dam, in Michigan (Table 1(*d*), Dam No. 49), the drain discharges at the toe of the ogee section, up stream from any jump that may form. In this case the drain may be down stream from the jump and discharge up stream, which would cause an unbalanced pressure acting downward on the apron, increasing its stability. Weep-holes that discharge directly through the apron equalize the pressure on the two sides, but they should be constructed in such a manner that the high velocities can not suck foundation material through them.

If it can be done conveniently, it is advisable to have drains discharge where the flow from them can be observed, in order that any failure to function properly may become known. In Northern climates, reliance should not be placed in weep-holes or drains that may freeze up and thus become inoperative.

By the use of weep-holes or drains constructed with reverse filters that prevent the removal of the foundation material, a reduction of the safe weighted distances given in Table 3 may be permitted. This assumes that the vents are: (1) Of sufficient capacity; (2) free from freezing; and (3) located a sufficient distance up stream from the lower end of the percolation path so that practically all the under-seepage will reach the tail-water through them. No fixed rule can be given on these requirements, but the exercise of reasonable judgment should prevent serious mistakes. For the location just up stream from the down-stream cut-off a reduction of 10% might be permitted, and smaller reductions could be allowed for less favorable locations.

LIMITATIONS OF CREEP ANALYSIS

As has already been stated, piping may occur as a result of percolation along the lines of contact of the dam with its foundations, or by flow through the foundation material itself. In a given case failure would result by travel along the path which offered the least resistance.

In order to determine experimentally the length of percolation path necessary to prevent piping from flow through the material, Griffith performed a series of experiments with a masonry trough $1\frac{1}{2}$ by $1\frac{1}{2}$ ft in section, filled with fine micaceous sand, and covered with earth, to determine the head of water necessary[25] to cause a blow-out. The results showed that a blow-out did not occur until the ratio of length of creep to head reached values of 3.33 to 2.5 although experience had shown that creep-head ratios of about $14\frac{1}{2}$ were necessary to secure safety in dams built on this sand. Griffith found that the flow of water through the sand column increased directly with the head until failure occurred, which came suddenly, without the previous formation of springs to form a channel and carry away the material. He also found that loading the sand decreased the tendency to blow out, and that water brought up suddenly on dry sand increased that tendency.

Much the same results were obtained by Professor Terzaghi[26], who found that the water percolating through the sand increased with the head until a point was reached at which the permeability of the sand suddenly decreased, and the velocity through it consequently increased, and a blow-out took place.

This increase in permeability is the result of a change of position of the sand grains to a less compact arrangement, giving them an increased bulk and greater voids. The effect of the added weight in Griffith's experiments was probably to resist this change of volume and, therefore, to increase the head necessary to cause a blow-out. This phenomenon has also been studied by Osborne-Reynolds[27], and by Warren J. Mead[28], Affiliate, Am. Soc. C. E.

Leliavsky has discussed a number of experiments to establish the head ratio at which blow-out occurs. In his paper[14] he explains the difference in the results obtained by various experimenters. This is due largely to the slope of the surface of the material where the water emerges. If the surface is inclined, less pressure is necessary to move the particles than if it is level, since gravity assists in the movement and a blow-out will take place, therefore, with lower head.

The data on the head necessary to cause blow-outs by flow directly through the material from actual dams are meager. The only dam in which failure unquestionably took place by flow directly through the foundation material is that at Coon Rapids, in Minnesota (Table 1(*b*), Dam No. 4). The tail-water level at this dam was so far beneath the apron that percolation could not occur along the line of creep, at least for part of the distance. This dam was on gravel and failed with a short-path ratio of 2.7. It is probable, however, that the surface where the blow-out occurred was on a slope, and, therefore, a blow-out occurred at a less head than would have been the case if the surface was level.

---

[25] Data from an unpublished part of Griffith's paper previously mentioned.

[26] *Technical Publication No. 215*, Am. Inst. of Min. and Metallurgical Engrs., p. 36.

[27] "Scientific Papers", by Osborne-Reynolds, Vol. II (1901), p. 217.

[28] "The Geologic Rôle of Dilatancy", *Journal of Geology*, Vol. XXXIII, No. 7, October–November, 1925.

The failures at the dam, in Port Angeles, Wash., and the Puentes Dam, in Spain (Table 1(*b*), Dams Nos. 1 and 2), may have been directly through the foundation material. In the former this seems especially likely. The situation in this case, however, was complicated by the piling driven just below the dam, which probably concentrated the flow and caused the blow-out (which took place with a short-path ratio of 1.9) to occur at a less ratio than would otherwise have been the case. The Puentes Dam failed with a short-path ratio of 1.4 and probably would have held at one of 2.4, although it had not sustained this ratio long enough to be certain.

It would seem that considerable light on this subject would be shed by experience with coffer-dams, as a great many of these have been constructed. When coffer-dams sunk in a saturated material are pumped out, water flows up through the bottom, tending to raise the foundation material and make it flow into the coffer-dam. If the water level outside the coffer-dam is at the same level as the ground surface, and the excavation in the coffer-dam is at the level of the bottom of the sheet-piling when the water inside the coffer-dam is pumped out, the head acting is approximately equal to the length of percolation path, making the short-path (as well as the creep) ratio practically 1.0. The length of path might be somewhat greater than the head, but this would probably be more than offset by the fact that the downward flow of the water outside the coffer-dam would take place through a much larger area than the flow upward into the coffer-dam. The condition would be that of flow through a channel of gradually decreasing cross-section and the velocity at the smaller end (in the coffer-dam), therefore would be greater than for flow through a uniform prism of material with a length equal to the head.

Two examples that have come to the writer's attention may be mentioned. In the case of a number of bridge pier foundations in the gravel of the Arkansas River bed at Pueblo, Colo., there was no tendency of the gravel to flow up into the coffer-dams. In a number of coffer-dams in Florida, with sand of the fineness of table salt, there was a considerable flow. These cases would indicate that a short-path ratio of 1.0 would be sufficient for gravel and not sufficient for fine sand. Griffith has mentioned a case of a coffer-dam in fine quartz sand with small stones that failed with a short-path ratio of 1.95. The writer hopes that discussers of this paper will supply instances which will throw light on this point.

The data are not sufficient to define closely the safe limit of length from the standpoint of flow directly through the foundation material. They do indicate in a rough way, however, that the assumption is justified that the flow takes place directly through the material when the distance is less than one-half the weighted-creep distance. On this basis one might assume that the limit of safe short-path distance would one-half that of safe weighted creep. There is another factor that enters, however: Even with well designed and maintained dams, scour may occur occasionally at the down-stream edge. Under these conditions the material on the side of the hole is more easily moved by percolating water than if it were on a flat surface. Moreover, the percolating water reduces the effective weight of the particles at the down-stream edge of the dam and makes them more subject to scour. For this reason, it is believed advisable to limit the short-path ratios to eight-tenths of those given for weighted creep. This limitation will seldom control the dimensions of the dam.

As previously stated, there is a defect in the creep analysis when rows of sheet-piling are too close together. Bligh recognized this and stated that the percolation followed the line of creep as long as rows of sheet-piling were not closer together than twice their depth.[29] Some such limitation of sheet-piling spacing is necessary because it is obvious that two rows of long sheet-piling very close together would offer little more resistance to piping than one row. If the rows were a great distance apart, however, they would offer considerably more resistance than one row. Bligh did not suggest any reason for using the limit of twice the depth rather than some other value; nor did he discuss what would take place if piling rows were spaced closer than his limiting value.

As the flow follows the path of least resistance, Bligh's limit seems to be equivalent to stating that when the piles are spaced at twice their depth, the resistance along the creep line is equal to that directly through the material, and when closer than twice their depth the resistance directly through the material is less than along the line of creep. When the lines of piles are spaced twice their depth apart, the creep distance between their lower ends is twice the short-path distance; and since for this condition the resistance along these two paths is equal, the resistance per unit length along the short path is twice that per unit length along the creep line. Wherever there are two points on the creep path so close together that a path exists directly through the foundation material which is less than one-half the weighted-creep distance between the points, the flow would take place directly through the material with a resistance twice that along the creep path. Where several overlapping short paths are thus possible, the one that gives the smallest combined weighted-creep and short-path distance for the entire structure is that on which the travel would take place.

For example, two lines of steel sheet-piling were driven 31 ft apart for the Middle Loup River Dam, in Nebraska (Table 1(*d*), Dam No. 48). One line extends 29 ft, and the other, 39 ft, below the bottom of the dam. One possible short path that fulfills the foregoing conditions extends horizontally from the end of the 29-ft piles to the longer ones. Other possible paths extend from various points on the shorter row of piling to various points along the longer row. The path extending between the lower ends of the sheet-piling rows gives the lowest combined weighted-creep and short-path distance for the structure of all the possible paths, and, therefore, should be adopted as the path to be used. When the water follow the short path, the path connecting the ends of the two adjacent shee piling rows will usually give the lowest weighted creep for the structur

---

[29] "The Practical Design of Irrigation Works", Second Edition, p. 168.

The foregoing reasoning applies also to the weighted creep. In this cas the horizontal creep is considered to have a weight of one-third, the vertic creep a weight of one, and the short-path creep a weight of two. In computin the plain and weighted creeps for existing dams, these rules have been use wherever applicable.

The rule that the flow follows the short path if its length is less tha one-half the weighted-creep distance between any two points in the cree path permits somewhat smaller pile spacing than Bligh's rule, since th horizontal creep, which, in Bligh's rule, has a weight of one in computin the creep distance, has a weight of only one-third in computing the weighted creep distance.

If boring records indicate a very pervious layer extending beneath th dam and coming near the surface at the down-stream toe, more conservatism in the short-path distance will be necessary because, as Professor Terzaghi has pointed out, this condition is conducive to high velocities where th water emerges and, therefore, danger of piping.

Upward Pressure

In masonry dams, upward pressure is important because of its tendenc to reduce the factor of safety against sliding. Sliding is not so importan a factor in the design of masonry dams on earth foundations. The principa danger from upward pressure on a dam on earth foundations is that it wil be sufficient to lift some part of the dam, breaking a new outlet opening fo the percolation which will give such a short percolation path that the da will fail. Upward pressure, therefore, is a factor in the failure o dams from piping.

As previously shown, dams may fail from piping by flow along the line of contact of the structure with its foundation, or by flow directly through the foundation material. In all dams some flow takes place directl through the foundation material, and no doubt the usual condition is that most of the flow occurs in this way. If a structure is designed so as to be safe against piping, the upward pressure under practically all of it will probably depend more on the conditions of flow through the foundatio material than along the line of contact. Even if a pipe did form, it would cover only a small area and the foundation masonry would bridge over it. Therefore, although it is necessary to give creep along the contact the pre-eminent place in determining the safety against piping, it may have less bearing in regard to upward pressure.

Under conditions in which the foundation material and its distribution are accurately known, it seems probable that some light on the upward pressure to be provided for, could be obtained with considerable accuracy by the flow net or electric analogy, although these results would be modified somewhat by the case of creep along horizontal surfaces. The writer hopes, n the near future, to investigate this further by a comparison of observed pward pressures on actual structures and the results obtained by the lectric analogy method. Where the foundation conditions are complicated r unknown, some approximate rule must be used. Measurement of upward ressure on actual dams and research by model experiment, flow net, and lectro-hydraulic analogy methods seem to indicate that the drop in pressure aused by a cut-off is greater in proportion to the creep distance than for the orizontal contacts. Leliavisky[30], after a study of the works of Coleman, orchheimer, Terzaghi, and Pavlovsky, states,

> "Thus, in calculating the hydraulic gradient the depth of the sheet-iling should be multiplied by a coefficient = 2, (sic) whereas there is lready sufficient evidence to show that this coefficient should be three or our or more."

In other words, the horizontal creep should be weighted three-fourths, ne-half, or less. Until more definite data are obtained, therefore, it is suggested that when it is necessary to use an empirical rule, the upward pressure e assumed to vary along the contact line in proportion to the weighted creep; (in other words, that the pressure drop along the vertical or steeply sloping contacts per unit of length be assumed to be three times that along horizontal contacts or slightly sloping contacts). This is believed to be a conservative rule as upward pressure experiments frequently indicate that a weight for horizontal creep of zero would give reasonably close results.

Where lines of sheet-piling are so close together that the flow may be considered to follow the short path between them rather than the line of creep, the upward pressure along the creep line between the ends of the short path may be considered to vary according to the weighted-creep distance between these points.

Special conditions sometimes cause unusual distribution of upward pressure. A dam founded on pervious material underlaid with impervious material and with a cut-off extending nearly to the impervious material, will have a lower uplift force down stream from the cut-off. A dam with such a foundation, and in which the cross-section of pervious material under the down-stream end of the apron is less than that at the up-stream edge, will have a greater upward pressure over the entire base than one founded on pervious material of great depth. This condition was noted in the upward pressure measurements on the Grand Valley Dam[31], in Colorado (Table 1(*b*), Dam No. 40). A contracting of the space through which the water flows beneath the dam acts as a partly closed valve; it raises the pressure up stream from it and lowers it down stream.

In computing upward pressure it should be remembered that the conservative assumptions may be different from those for safety from piping. In the latter case, the creep under flexible aprons and similar construction down stream from the solid part of the dam, should be neglected in estimating the security of the dam against piping, as their effect is uncertain and unreliable. In computing the upward pressure they should be give consideration, however, as they may cause greater upward pressure to occu at the down-stream edge of the solid part of the dam than the other assump tion would indicate. For example, the upward pressure experiments on th Pinhook Dam[32], in Iowa (Table 1(*b*), Dam No. 22), showed a considerabl upward pressure at the down-stream edge of the solid part of the dam due no doubt to the flexible apron down stream, although if the ordinar assumption for the design of such structures for safety against piping wer used in determining the position of the end of the creep path, there would be no upward pressure at that point.

---

[30] "On Percolation Under Aprons of Irrigation Works", p. 46.

[31] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1532.

At this point it is worth while to suggest that the upward pressure b assumed to be applied at the bottom of the foundation; it really is so applied an has a magnitude equal to the difference in elevation between this poin and the piezometric line. The total weight of the masonry (in air) an of any water which may be above it, can then be considered as resisting thi pressure. This will eliminate any uncertainty as to whether or not th masonry ought to be considered as submerged (and, therefore, as having los weight) and will prevent a mistake sometimes made, of measuring the upward pressure by taking the difference between the elevation of the top o the apron and the piezometric line, while still assuming the full weight of the masonry available to resist it. Experiments by H. de B. Parsons M. Am. Soc. C. E.,[33] and others, have shown the necessity of assuming that the upward pressure acts over the entire area of the base of the dam.

Bligh advocated a factor of safety in determining the thickness of the apron required to resist the upward pressure, by making it heavy enough to resist four-thirds of the computed upward pressure. It would be desirable to analyze all existing upward pressure measurements to fix more accurately what factors of safety are required, since there is considerable evidence that the pressures will be less than computed either by the Bligh, or by weighted-creep, rules. Until further information is available, however, it is believed that the factor of safety of four-thirds used by Bligh should also be used with the weighted-creep rule.

CONCLUSIONS

The ordinary method of analyzing a masonry dam on an earth foundation to secure safety against piping, which is usually ascribed to Bligh, is faulty in that it does not consider the greater probability of percolation along level or slightly sloping contacts between the dam and its foundation, than along vertical or steeply sloping contacts. The flow-net and electric-analogy methods are faulty in neglecting the lesser resistance along the line of contact of a dam with its foundation as compared with that directly through the foundation material.

Piping may occur in two distinct ways: (1) By flow along the line of contact of the structure and its foundation; or (2) by flow directly through the foundation material. Flow ordinarily occurs along both these paths in nverse proportion to their relative resistances. Considerable light on the robability of failure from the first of these causes can be obtained by xperimental methods, but the second cause must be studied largely by an nalysis of the action of actual dams.

---

[32] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1551.

[33] *Loc. cit.*, p. 1317.

From the result of an analysis of the action of more than 200 dams, it as been found that creep along contact surfaces having slopes with he horizontal of less than 45° should be considered to offer only one-third the esistance to piping as those with slopes of 45°, or more.

Analysis on this basis may be called the weighted-creep analysis, the reep along surfaces that slope less than 45° being termed "horizontal creep" nd that of 45°, or more, "vertical creep". It should be noted that these lopes are those of the surface of contact. The slope of the path taken by he water of a dam may be different. The weighted-creep distance of a cross-ection of a dam is the sum of the vertical creep distances plus one-third he sum of the horizontal creep distances, and the weighted-creep-head ratio s the weighted creep divided by the effective head.

A schedule of safe weighted-creep-head ratios for use in the design of ajor structures is given in Table 3. These values can only be used for solid masonry cut-offs built directly against the earth, or for interlocking steel or concrete sheet-piling driven so that the interlock is not broken and with the top of the piling satisfactorily embedded in the masonry of the dam. Competent supervision during construction and efficient maintenance are assumed. If all these do not exist higher values must be used. For less important structures these ratios may be reduced somewhat, to perhaps 80% of the values given, for very minor structures.

Reverse filters, weep-holes, and drains are aids to security, and weighted-creep-head ratios may be reduced as much as 10% if they are used. The best form is a weep-hole with a reverse filter behind it. For best results vents should be far enough from the end of the travel path to insure that most of the flow passes through them. Except to reduce upward pressure, they should not be farther up stream than necessary to accomplish this purpose. Usually, the best location, from the standpoint of piping, is just above the down-stream cut-off, venting through the cut-off. The position of the hydraulic jump should be considered in locating vents. Great care must be exercised in constructing vents or drains.

In all cases care must be exercised to insure that cut-offs are properly tied in at the ends, so that the water will not outflank them, and that there is no short route behind or under the abutments through which a channel may be formed.

In order to prevent failure of dams by percolation directly through the foundation material the short-path-head ratios should not be less than eight-tenths of those recommended for the weighted creep.

If any two points on the creep line are so close together that the short path between them is less than one-half the weighted-creep distance between them, flow may be considered to take place directly through the material, the length of this travel being given a weight of two. Where more than one short path is possible between the same parts of the creep path, the one tha gives the smallest total weighted creep for the structure should be used.

The upward pressure to be used in design may be estimated by assumin that the drop in pressure from head-water to tail-water along the conta line of dam and foundation is proportional to the weighted-creep distanc Between sheet-pile lines that are so close together that the short-path limitations apply, the total pressure drop may be computed as proportional to th short-path distance with its weight of two and may be distributed between th two ends of the short path in proportion to the weighted creep betwee these points.

ACKNOWLEDGMENTS

It would have been impossible to have collected the amount of dat analyzed in this paper without the hearty co-operation of a large number o engineers. The sources of data on the individual dams is acknowledged i full in the original manuscript. Other acknowledgments are made in number of places in the text. In addition, the writer wishes to express hi indebtedness to the following:

To the Ambursen Dam Company for the data on a large number of dam of the Ambursen type, on earth foundations, with most of which tha Company was connected as designer or contractor; to Messrs. Ayres, Lewis Norris, and May, for data on dams designed by them, or by the lat Gardner S. Williams, M. Am. Soc. C. E.; to Mr. W. M. Griffith for man valuable suggestions and ideas and for data on several dams; to Messrs Holland, Ackerman, and Holland for data on a number of dams designe by them; to the Department of Public Works, Philippine Islands, for dat on dams built by it; to the Verein Deutscher Ingenieure and the Deutsch Wasserwirtschafts and Wasserkraftverband for assistance in securing information on failures in Germany; to the U. S. War Department and it various District Offices for data on dams built by the U. S. Corps o Engineers; to D. W. Mead, Hon. M. Am. Soc. C. E.; A. L. Alin, H. E Babbitt, J. L. Burkholder, E. L. Chandler, C. H. Eiffert, B. A. Etcheverry D. C. Henny, L. C. Hill, Ivan E. Houk, R. D. Goodrich, John H. Gregory Fort Kurtz, R. B. McWhorter, and the late William W. Tefft, Members Am. Soc. C. E.; to George R. Clemens, Assoc. M. Am. Soc. C. E.; to Messrs P. Bier, G. P. F. Boese, H. N. Eaton, W. H. Frazer, Charles G. Gablemen C. C. Neher, Paul A. Nelson, A. D. Williams, and numerous others, fo assisting in securing data and for valuable suggestions. The writer was assiste in preparing the data and drawings by Messrs. Walter H. Price, W. F Bingham, and Richard J. Willson.

This paper represents the result of a study made for the U. S. Burea of Reclamation to assist in the design of an intake dam on the Colorad River for the All-American Canal. All the work of this Bureau is under the direction of Elwood Mead, M. Am. Soc. C. E., Commissioner o Reclamation. All engineering work is in charge of R. F. Walter, M. Am Soc. C. E., Chief Engineer. The study was carried out under the immediate supervision of J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

UPLIFT AND SEEPAGE UNDER DAMS ON SAND

By L. F. Harza[1], M. Am. Soc. C. E.

Synopsis

The basic principles of flow under dams on sand are presented briefly in this paper in the hope of stimulating scientific study of this much neglected subject. Analytical methods applicable to some familiar types of foundations, and graphical and electric analogy methods of general application are described, by which can be determined the theoretical laws governing: (1) The hydrostatic pressure along the foundation contact; (2) the hydraulic gradient with which the water escapes upward at the toe; and (3) the approximate leakage under the structure. These theoretical laws are to be used as scientific nuclei around which to build future experimental data and field observations to the end that the order of magnitude of field variations from scientific principles and the safe coefficients or factors of safety for practical applications may be determined.

Introduction

The problems encountered in the design of a masonry dam on a sand foundation are:

(1) To provide safe conduct for seepage water under and around it, so that neither foundation nor bank material will be removed.

(2) To reduce seepage losses under and around it to a reasonable and economic minimum, consistent with construction economy and the value of wasted water, safe conduct being assumed under Problem (1).

(3) To obtain stability of structures under all conditions of loading, uplift, overflow, buoyancy, tail-water fluctuation, etc.

(4) To design the structure so as to distribute the load and to minimize settlement, and to permit it to accommodate itself to unavoidable settlement so as to prevent internal stresses and cracking.

Note.—Discussion on this paper will be closed in December, 1934 *Proceedings.*

[1] Cons. Engr.; Pres., Harza Eng. Co., Chicago, Ill.

(5) To destroy the energy of waste water at the toe of the spillway in such a manner as to avoid injury to the apron and to minimize the erosion o stream bed and banks.

(6) To anticipate, in the design of all structures, the possible limi of degradation of river channel below the dam and consequent subsidence o tail-water level as affecting (*a*) undermining and stability of structures; (*b*) functioning of spillway; and (*c*) loss of draft head (if hydro-electric).

Problems (5) and (6) are related only to spillway structures and will not be considered herein because of prescribed space limitations of this paper. Problems (1) to (4) require a knowledge of: (1) Uplift pressure; (2) upward escape gradient at the toe; and (3) seepage volume. This paper will be confined to a treatment of the fundamental principles of determining these conditions, indicating methods of investigation and study, but for the most part omitting illustrative examples of their application.

Bligh's Empirical Coefficient.—Modern practice in the design of dams on sand was developed to a great extent from experience in India, as elucidated by Mr. W. G. Bligh[2]. His empirical coefficient, *C*, now generally called the "percolation factor" or "coefficient", is the ratio of the percolation distance of the water passing under the dam to the applied head. It is thus the reciprocal of the average "hydraulic gradient". For four classes of material, Bligh recommends the percolation factors given in Table 1. As the percolation distance, Bligh uses what is sometimes called the "line of creep", defined as the total length of contact between structure and sand, from head-water to tail-water. This assumes that seepage will follow along the bottom of a structure until sheeting or some other cut-off wall is encountered; thence downward along and under this cut-off; thence upward on the down-stream side of the cut-off and again along the bottom of the structure. In lieu of the "line of creep", the "short path" distance of flow is sometimes used, which defines itself.

TABLE 1.—RECOMMENDED PERCOLATION FACTORS

Class	Description of material	Coefficient, C
1.......	River beds of light silt and mud, such as that of the Nile..	18
2.......	Fine micaceous sand, as in the Himalayan rivers and in such rivers as the Colorado in the United States........................	15
3	Coarse-grained sands, as in Central and South India (this is the most common type)	12
4.......	Boulders or shingle and gravel and sand mixed........................	9 to 5*

*Varies

The Bligh coefficients were based upon meager data as to failures, and upon only superficial descriptions of the foundation materials. They were derived from observations of the behavior of low spillway weirs consisting frequently of loose rock-fill between longitudinal dividing walls. This type of weir was scarcely more than a well-paved, sloping stream bed where the erosion of the dam and the stream bed was perhaps the primary consideration. These coefficients have since been broadly applied to non-overflow and massive types for which they were not intended. The reliance which has thus been placed upon them is an indication of the primitive state of the art and of the tendency of the profession to lean upon precedent and rules after they have once been codified, as by Bligh. The writer feels that this, perhaps, too definite statement of coefficients has long served as a deterrent to further progress in the study of this problem. The man who does not use what others have used before him must accept more responsibility as the price of his initiative.

---

[2] "The Practical Design of Irrigation Works", by W. G. Bligh, D. Van Nostrand Co.

The writer hopes to reveal some basic principles which may help to establish a more scientific approach to the problem. He is fully aware that, if such approach is based upon the assumption of homogeneous foundation material, the conclusions may not be closely paralleled in practice. If based upon the best available data as to variations in material, they will at best be only rough approximations because of the difficulty and cost (if not impossibility) of obtaining accurate knowledge by borings, wells, pumping tests, and other available methods, of complex natural deposits. Nevertheless, a knowledge of the laws of underflow in uniform material is valuable in revealing basic principles. Such knowledge will form a basis for intelligent modification by the choice of a factor of safety dependent upon the degree of non-uniformity, and upon the extent of uncertainty as to existing conditions. Before attempting, however, to develop a rational rather than a purely empirical basis for the design of dams on sand, it is necessary to inquire rather minutely into the principles of the hydrodynamics of soil, and the forces exerted by seeping water.

Darcy's Law.—The principle stated by Darcy[3] that the velocity of seepage is proportional to the hydraulic gradient has been long the fundamental basis of attack, many times verified, for all types of granular material. Thus, with a quantity, q, flowing through a gross sectional area, a, with net effective velocity, v (applied to the gross section), through a length of path, l, under a head, h:

$$v = K \frac{h}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and,

$$q = a v = a K \frac{h}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In Equation (2), K is the net effective velocity of seepage (as through the gross section rather than the pore section) through a volume of the material of unit length subjected to unit head; or it may be expressed as the velocity under a hydraulic gradient of unity, and is known as the "'transmission constant". The constant, K, is dependent upon temperature, effective size and shape of grains, density, type of packing, and porosity. It has been investigated by the late Allen Hazen,[4] M. Am. Soc. C. E., and by Professor C. S. Slichter.[5]

---

[3] "Les Fontaines Publiques de la Ville de Dijon", par H. Darcy, 1856, p. 590.

[4] "Rept., Mass. State Board of Health, 1892.

[5] *Water Supply and Irrigation Papers Nos.* 67 and *140*, U. S. Geological Survey.

General Principle of Pressure Due to Flow

Flow through a granular material can only occur as a result of, and in the direction of, a decrease of hydrostatic head as between two points in the material; and such decrease in head along the path of flow exerts a force upon this material, in the direction of flow, in the form of a higher pressure on the approaching side than on the receding side of each grain of material. Obviously, if the hydraulic gradient is unity the differential pressure on each particle will be equal to its displacement. For greater or smaller gradients the pressure exerted on each particle is proportionately greater or less than the displacement. The action is also complicated and increased by hydraulic friction and is of sufficient importance to analyze in detail. Consider some simple examples to illustrate the principle.

Pressure on Bottom of a Cylindrical Vessel.—Let s = specific gravity of sand grains = about 2.65 average value; w = unit of weight of water; P = percentage voids in the material; $\Delta = w\ (1 - P)$ = displacement of one cubic unit of material; $w\ s$ = weight of one cubic unit of solid material; $w_s = w\ (s - 1)$ = submerged or buoyed unit weight of solid material; and $w_l = w\ (s - 1)\ (1 - P)$ = buoyed unit weight of loose material.

If sand and water are placed in a cylindrical vessel with a solid bottom, the pressure, p, on the bottom will be: (1) The independent weight of the contents; or it may be conceived as produced by, and equivalent to, (2) that of hydrostatic pressure transmitted undiminished through the sand and acting over the entire bottom area, plus the submerged or buoyed weight of sand. These conceptions can be expressed by the following algebraically equivalent equations, respectively (see Fig. 1):

$$p = wah_1 + waPh_2 + wah_2s\ (1 - P) \dots\dots\dots\dots\dots\dots (3)$$

and,

$$p = wa\ (h_1 + h_2) + wah_2\ (s - 1)\ (1 - P) \dots\dots\dots\dots\dots (4)$$

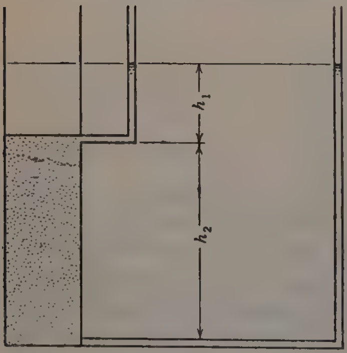

FIG. 1.

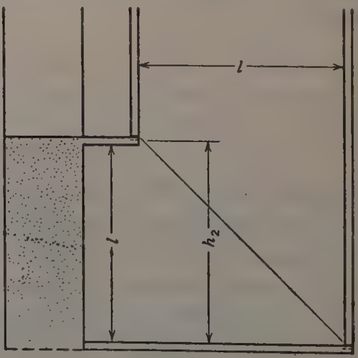

FIG. 2.

Cylinder with Screened Bottom.—If the bottom of the cylinder is screened instead of solid, one assumes water to be supplied at the top of a cylinder at the same rate as it escapes by seepage from the bottom, thus maintaining constant conditions. It is obvious that the load on the screened bottom must remain equal to the contents of the cylinder as on the solid bottom; and yet no part of this load is transmitted to the bottom by static hydraulic pressure as before. It is exerted entirely through the solid material, the water acting only to increase the effective weight of the particles and thus to increase the pressure on the screen of the bottom-most particles in contact with the screen.

Thus, Fig. 1 represents the solid bottom with piezometers both standing level, each sand grain being acted upon by buoyancy. Fig. 2 shows a screened bottom with the entire head, h_2, lost in friction through the sand, and with $h_1 = 0$ (except for a slight film of water to prevent capillary action). Then, the loss of head is 1 ft per ft of sand column, or just enough to overcome buoyancy; that is, the hydraulic gradient is 1.0. There is no static water pressure anywhere in the column, and, therefore, the pressure on the top and bottom of each sand grain is zero, indicating that there is neither buoyancy nor downward static pressure acting on the sand grains. The sand load, therefore, is its own dry weight plus only the hydraulic friction of the seepage flow. This friction, which results from a gradient of unity, must, therefore, equal the weight of pore water, $= wah_2P$, if the total pressure on the screen is to remain equal to the pressure on the solid bottom. This force seems to be partly of the same nature as the forward thrust of flowing water caused by friction with the wall of a pipe; or, it may be contributed partly by the excess dynamic force on the approaching side necessary to deflect the flow around each particle, and the corresponding deficient pressure on the down-stream side.

It is now evident that the forces exerted by water flowing through sand are composed of three elements: (1) A static differential pressure on the approaching and receding faces of each grain, resulting from the reduction in pressure along the route of flow and acting in the direction of flow; (2) a frictional or dynamic force acting in the same direction; and (3) buoyancy, acting upward and unrelated to the flow. For a gradient of unity, the differential static heads on the approaching and receding faces of each particle differ by the mean thickness of the particle, and, therefore, the force exerted is equal in amount to the displacement of the particle. Furthermore, the frictional-dynamic force is equal to the weight of the pore water, as previously proved, and the total force, therefore, is equal to the weight of water necessary to fill the total volume of the cylinder. With the hydraulic gradient greater or less than unity, obviously, the differential static pressures on each grain will increase or decrease in like proportion, and to maintain the accuracy of Darcy's law the frictional-dynamic forces must also vary in like ratio, the two combining to form the "gradient pressure".

The conclusion is justified, therefore, that under all conditions, the increment force, in addition to buoyancy (or "gradient pressure"), exerted by seeping water against a volume of granular material will be the difference in hydraulic head at the approaching and receding faces of the volume applied to the entire cross-section, as on a solid, instead of granular, piston, although the pressure is actually applied gradually through the material instead of against an impervious face. The aforementioned principle can be demonstrated mathematically in a much briefer manner, but the foregoing dissected method is chosen to give a clear conception of what transpires in the pores of the material. A few illustrations will further clarify this point.

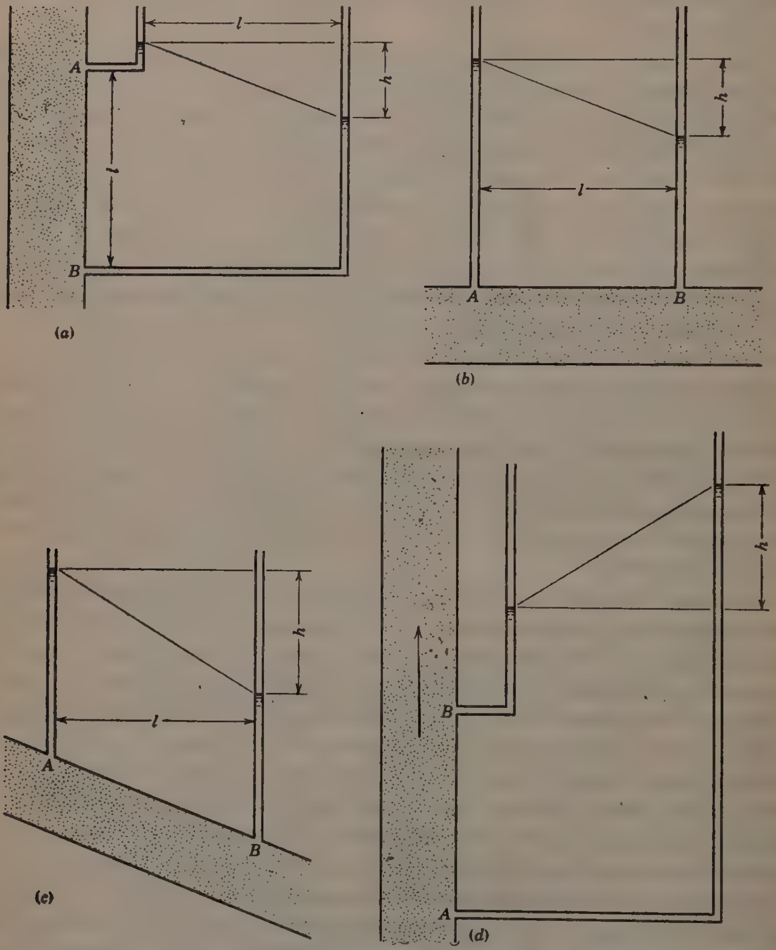

FIG. 3.—THE GENERAL CASE OF SEEPAGE THROUGH A SAND COLUMN.

In the general case (Fig. 3) of seepage through a sand column of Cross-Section a, the total differential force between Points A and B resulting from movement of the water will be wah. This must be added vectorially to the buoyant weight of sand, $wal\ (s - 1)\ (1 - P)$, to obtain the total resultant force. In Fig. 3(a) the seepage force and buoyancy act opposite; in Fig. 3(b), normal to each other; in Fig. 3(c), at a variable angle; and in Fig. 3(d), in the same direction.

Quicksand, Sand Boils, Piping, "Flotation."—The most important case in its bearing upon the safety of a dam founded on sand is that of Fig. 3(*d*), in which water is flowing upward through the material. Here, the force of flow or "gradient pressure" combines with that of buoyancy to reduce the effective weight of the material and will actually float the material if the internal hydrostatic pressure of the seeping water becomes equal to, or greater than, the load of superimposed material. This critical condition occurs when:

$$wah = wal\ (1 - P)\ (s - 1) \qquad (5)$$

and the ""flotation gradient" is:

$$F_c = \frac{h}{l} = (1 - P)\ (s - 1) \qquad (6)$$

Charles Terzaghi,[6] M. Am. Soc. C. E., has studied this phenomenon by platting a curve between the quantity of percolation upward through a cylinder of sand and the hydraulic gradient or "escape gradient" required to produce it. He finds a sudden increase in percolation when the critical or "flotation gradient" is reached (see Fig. 4) and explains it by calling attention to a sudden swelling of the sand into a more porous or "quick" condition. "Quicksand" is thus primarily a condition characterized by the flow of seepage water upward through the sand, rather than a characteristic of the material itself; and, yet, it is customary to associate the term, "quicksand", only with fine sand. This is the case because fine sand when once expanded with an excess of pore water and consequent voids, retains this condition for some time even with the removal of pressure because of the greater resistance to, and time required for, escape of the contained water through the small pores. Any granular material, such as coarse sand or gravel, may become "quick" with a sufficient supply of water, and under sufficient pressure, to satisfy the critical equation. The jetting of piles is merely the artificial creation of a "quick" condition to make the sand more fluid.

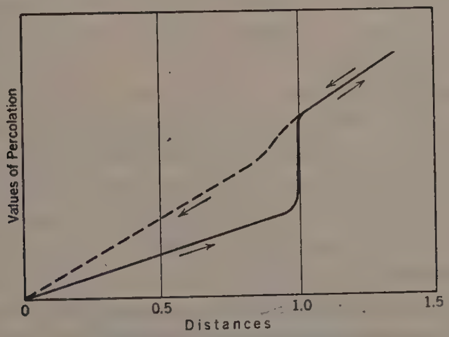

FIG. 4.—UPWARD GRADIENT.

---

[6] *Technologic Publication No. 215*, Am. Inst. of Min. and Metal. Engrs., p. 36.

Flotation at Toe of Dam.—Fig. 5 is a summary of the forces acting on a unit volume of sand along the route of seepage under a dam, exclusive of the load of the structure. In this diagram, G = the gradient or hydraulic thrust $= \frac{h}{l}$; W_s = the submerged weight of sand; and R = the resultant of G and W. It will be noted that the water entering the foundation sand on the up-stream side of the dam adds to the effective weight of the sand, and tends to compact it. This increases its resistance to seepage at entrance which is also increased by the formation of a filter skin. Under the middle of the dam the seepage causes a horizontal load on the sand while the buoyed weight is downward, and the resultant inclined. On the down-stream side of the dam the water rises to the surface and its gradient pressure tends to combine with the buoyancy to reduce the effective weight of the sand. From Fig. 5 it is evident that the sand is in a stable condition throughout the route of seepage, except as it rises to escape at the toe. Under critical conditions, therefore, there is no similarity to imminent pipe flow and the word, "piping", is not descriptive, but is actually misleading. The word, "flotation", is descriptive of this critical condition at the toe, and will be used herein. The "escape gradient" is vertical at the surface and tends always to reduce the effective weight of the material. If it reaches the critical value, or "flotation gradient", actual flotation results. The sand swells, causing an increase in porosity, and perhaps flows away with escap-

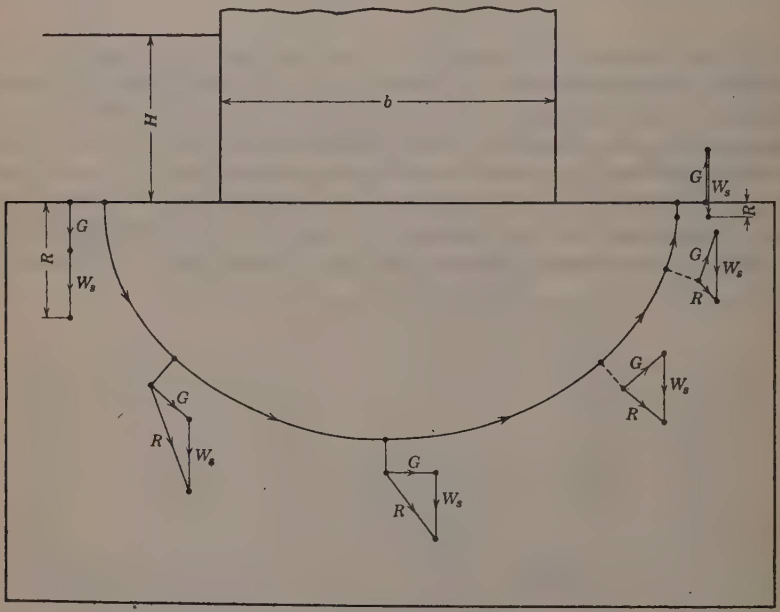

FIG. 5.—FORCES ACTING ON A UNIT VOLUME OF SAND ALONG THE ROUTE OF SEEPAGE UNDER A DAM.

ing water. Thereafter, the resistance to escape is further decreased which may further increase the flow and escape of sand. Thus, the sand may crumble back from the point of escape until a channel is progressively opened from tail-water back to head water under the dam. This result is evidently responsible for the word, "piping", and for the misconception of the nature of the phenomenon which the word implies. "Flotation" is the cause and "piping", the effect. If "flotation" is prevented, piping cannot occur. Therefore, a designer is concerned as to safety only with the conditions obtaining at the point of escape of the seepage water.

Uncertainties enter into the computation of the critical conditions of flotation at the toe because cross-flow may occur by convergence toward the easiest point of escape where the material is non-homogeneous, and the hydraulic gradient, $\frac{h}{l}$, at the point of escape is seldom equal to the average gradient, $\frac{H}{L}$, even in homogeneous material. Furthermore, if streaks, lenses, or strata of porous material are interbedded with finer material, the relative permeability may encourage concentration of flow from the finer into the coarser material near the outlet, giving the latter more than its proportionate quantity and a higher rate of loss of head at the point of escape. It is the function of the factor of safety, if not otherwise discovered and provided for, to allow for such conditions. This has been discussed by Professor Terzaghi under the subject of "Minor Geologic Details"[7], although it is often of major importance to the success of the structure. The existence of such "minor details" does not alter the origin of the failure as that of toe flotation. It does add to the difficulties of predicting the local flotation gradient. In fact, as stated by Professor Terzaghi[7], often such "features can be predicted neither from the results of careful investigations of a dam site nor by means of a reasonable amount of test borings".

Considered as a purely laboratory problem, the determination of critical or flotation gradient for a given material is reasonably definite once the characteristics of the material are known. In Professor Terzaghi's experiments on varied materials[8] the measured internal pressure to cause flotation always equalled or exceeded the theoretical, by ratios varying from 1.00 to 1.24. Thus, in general, the computed critical pressure apparently is on the safe side, and no factor of uncertainty need be introduced except to the extent that the relation of the escape gradient, $\frac{h}{l}$, to the known dimension of the structure is unknown and in so far as the characteristics of the sand and its stratification are unknown or incapable of interpretation in terms of the gradient at the point of escape of the water.

---

[7] *Technical Publication No. 213*, Am. Inst. of Min. and Metal. Engrs., p. 31.

[8] "Erdbaumechanik", p. 132.

CRITERIA OF DESIGN

The three conditions requiring determination in order to design or test a dam on sand are: (1) The uplift pressure under the base of the dam, as related to its weight; (2) the vertical escape gradient at the toe as related to the physical characteristics of the material; and (3) seepage volume, although not mathematically related to safety.

In all mathematical or experimental investigations of base uplift pressure or flotation gradient at the toe, in homogeneous material, the transmission constant, K (Equation (1)), cancels out. Neither Criterion (1) nor Criterion (2) is dependent upon quantity or velocity. Regardless of the size and porosity of the material, there is the same total head to dissipate in the same distance and, therefore, it will be dissipated at the same rate in any homogeneous material; the quantity of water that escapes will be directly proportional to the transmission constant; the water will escape with the same toe gradient and will cause the same uplift pressure under the base. Thus, in homogeneous material, the amount of leakage is largely an economic problem. Even a foundation of large boulders with no finer material might be safer with a large leakage than finer material with negligible leakage, or *vice versa* A method of estimating leakage will be introduced later.

Graphical Analysis.—Forchheimer has developed a very ingenious graphical method[9] of platting the lines of flow and equal pressure lines conveniently called a "flow net", illustrated in Fig. 6, and based upon flow in a plane normal to the axis of the dam. The same quantity of water is assumed to flow between each pair of consecutive flow lines. The velocity, therefore, must be inversely proportional to the spacing of these lines at each point. Since the drop in pressure is directly proportional to the velocity, the spacing of the equal pressure lines must everywhere bear a constant ratio to the spacing of the flow lines at that point. It is convenient to make the spacing of the two systems of lines equal and each figure an approximate square with the same average breadth and length.

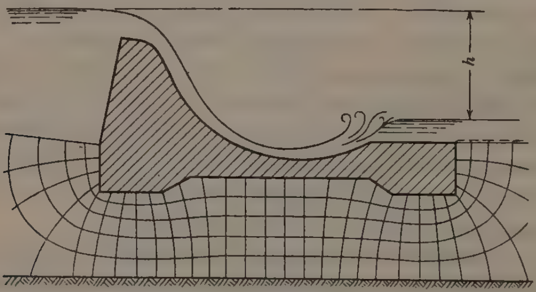

FIG. 6.—TYPICAL "FLOW NET" OR LINES OF FLOW AND EQUAL PRESSURE.

Since direction of flow is in the direction of drop of pressure, the lines must everywhere intersect at right angles. With these principles in mind the problem becomes purely geometrical, a process of successive approximations, or "trial and error". Obviously, the first flow line must coincide with the contract line of the structure with the sand. The last line must coincide, or must be parallel with, the rock line. On attempting to plat such a system of lines it will soon be discovered that they must be spaced closely at the protruding corners of the structure, and at the tip of the sheet-piling; otherwise, obvious distortion and reverse curvature and non-parallelism with the rock will result.

---

[9] "Hydraulik", von Ph. Forchheimer, 1930, p. 82.

Some practice will be required to make reasonably close assumptions as to the spacing of the first strip of squares adjacent to the structure. As the network is then built downward and outward, if inconsistencies develop, it is necessary to re-assume the spacing of the first row and proceed again until consistent curves are obtained. The process is a tedious one, but is mathematically sound within the limits of accuracy of the approximation of the small distorted figures to true squares. Strata of assumed greater or less permeability can be introduced by this method, involving complications which would defy mathematical analysis[10].

The Hydraulic Electric Analogy.—The fact that Ohm's law for flow of electricity, $\left(C = \frac{E}{R} = K' \frac{E}{l} a\right)$, is identical in form with Darcy's law of flow of water in granular material (see Equation (2)) permits the solution of underflow problems, by electric analogy utilizing the principle of a Wheatstone bridge. The writer's acquaintance with this principle was first obtained from E. W. Lane, M. Am. Soc. C. E.[11] Other investigators have platted complete "flow nets"[10] which are very fascinating and instructive, but the writer has chosen to determine only the three important criteria in order to economize time and cover the greatest number of conditions in a first survey of the field.

The writer's "tray" is connected somewhat differently from that of previous investigators, as shown in Fig. 7. It is 24 by 46 in. in size, utilizing a salt solution about 1 in. deep. Copper terminals, X and Y, permit simulating the base of a dam, 20 in. or less in width, A B, of any desired sectional design. With Switch E closed to F, the earphones, P, permit the determination of the proportionate voltage (or hydrostatic uplift head) drop at C along the base by selecting Point D to produce silence. Likewise, to determine the resistance of the solution between the copper strips, X and Y, Switch E is closed to F' through the known resistance, R, with earphones connected to C'.

To obtain unit resistance of the solution for parallel flow the two main terminals are changed from the copper strips, X and Y, to X' and Y', in a bakelite frame temporarily immersed; or the copper strips, X and Y, are replaced by strips along the entire ends of the tray while the model of the dam base is removed. Knowledge of the exact depth of the solution is not necessary and would be difficult to determine in such a shallow tray and likewise would be subject to frequent change by evaporation and replacement. It is only necessary to know the resistance between the terminals, X and Y, compared with the resistance producing parallel flow, as between X' and Y', in an equal depth of solution.

---

[10] *Technical Publication No. 215*, Am. Inst. of Min. and Metal. Engrs., p. 39.

[11] *Civil Engineering*, October, 1934.

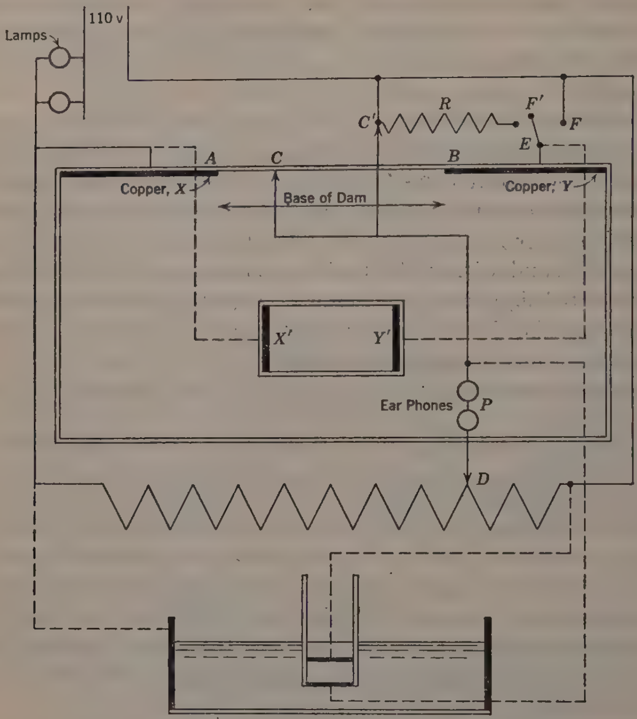

FIG. 7.

Quantity of Seepage.—In order to estimate the quantity of seepage to be expected under a given type of foundation, it becomes necessary to correlate the tray analogy with the physical characteristics of the material in the field. There is no present method known to the writer for determining the transmission constant, K, for sand in place except by Slichter's method of measuring velocity of underflow combined with test pipes for measuring slope. The pumping of a test well or test pit does not now give K because the information thus gained is not interpretable into K by any known law, due to the complex lines of flow converging through the material in three dimensions toward the outlet. The hydraulic electric analogy offers a rough basis for this solution. The side wall of a tub (Fig. 7), 18 in. in diameter was lined with copper and the tub was partly filled with salt solution. A piece of electrical fiber conduit was partly submerged in the center. This conduit was fitted with one circular metal disk that nearly closed the bottom and one close-fitting disk, adjustable in height above the bottom, both disks being connected to exterior binding posts by insulated conductors.

Without further detail it is evident that the Wheatstone bridge in Fig. 7 permitted the measurement of the relative resistance of the three-dimensional converging paths of flow toward the lower disk as compared with the resistance between disks within the tube representing parallel flow through the same solution. It was determined that the path between the two disks within the conduit, in order to offer the same total resistance as the converging flow from tub wall to lower disk, needed the disk spacing listed in Table 2, Column (3).

TABLE 2.—DISK SPACING

Diameter of duct, in inches (1)	Height of adjustable disk above bottom, in inches (2)	Distance between disks, in inches (3)	Ratio of spacing to tube diameter (Column (3) divided by Column (1)) (4)
2	2	0.6	0.3
2	4	0.56	0.28
4	4	1.32	0.33

Nearly all the converging resistance is encountered close to the entrance where current density becomes high, and, therefore, the diameter of tub and the depth of water can be shown to have little effect. It may be stated, therefore, that within practical limits of accuracy of application, the draw-down head required for a given water yield of a test pit would discharge the same quantity of water longitudinally through a volume of the same material having the same cross-section as the test pit and a length equal to three-tenths of the test-pit diameter. Thus, one test pit, 6 ft in diameter, yielded 900 gal per min, or 2 cu ft per sec for a draw-down of 4 ft. For parallel flow, then, by Equation (2): $q = 2 = K \frac{4}{0.3 \times 6} 28$, or $K = 0.0322$ ft per sec, or 1.932 ft per min.

It is equally possible to mount metal tubing on bakelite rods with a varying ratio of length of tube to its diameter and then to submerge them in the same salt solution to obtain resistance of convergency to such a surface. This simulates a screened driven well, or well-point, as compared with the resistance to parallel flow, thus permitting determination of K by a similar analogy. The writer's data thus determined are too meager to report because this principle does not admit of as simple a statement as the test pit. The pumping of test pits and of driven well-points, or screened wells, thus furnishes, with the electric analogy, a basis for estimating the total seepage under any type of foundation design.

Thus, the resistance from X to Y (Fig. 7) under the model of a certain proposed design of power-house foundation was 7.6 ohms, while the resistance from end to end of the 24 by 46-in. tray was 9 ohms. The scale of the model was 25 ft to the inch. The parallel flow through a prototype section of the tested material, 1 ft thick, under a 65-ft head, would be, by Equation (2), $q = 0.0322 \frac{65}{46 \times 25} 24 \times 25 = 1.09$ cu ft per sec, in which it will be noted that the scale of the model is cancelled. The flow under the proposed power house would then be $\frac{9.0}{7.6} \times 1.09 = 1.29$ cu ft per sec per foot length of the structure.

This computation ignores depth to rock or to impervious material. If known, the tray can be partitioned horizontally to simulate this depth; if not, it is believed the error will not be great and that usually it will be within the limits of accuracy of the method, as most of the seepage flow hugs close to the foundation profile of the structure.

ANALYSES OF SPECIAL CONDITIONS

Dam with Base Flush with Stream Bed, No Sheeting.—Weaver[12] has analyzed, mathematically, the problem of uplift pressure on the base for the case of a dam on homogeneous sand of indefinite depth, with the base of the dam resting on, and level with, the sand surface, with and without a single line of sheet-piling. He does not treat the equally important problem of escape gradient of the water at the toe. Weaver finds, under these simplified assumptions, with no sheet-piling, that the paths of flow are the lower halves of a family of confocal ellipses with foci at the heel and toe, and the lines of equal pressure are the conjugate family of confocal hyperbolas. In Fig. 8 the paths of flow are shown as full lines and the lines of equal pressure as dotted lines. Weaver has shown that the time-honored assumption of straight-line uplift gradient from heel to toe is fundamentally incorrect in principle, although not seriously so, and on the safe side. He finds that the uplift is given by the formula[13]:

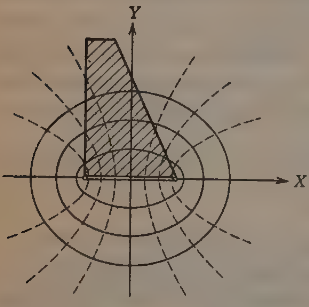

FIG. 8.—CONJUGATE ELLIPSES AND HYPERBOLAS.

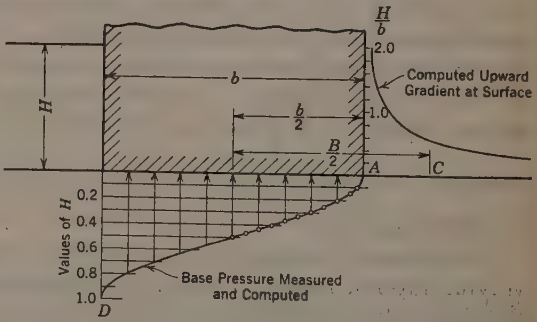

FIG. 9.—DAM ON SAND—NO CUT-OFFS.

$$\text{Uplift head} = \frac{H}{\pi} \cos^{-1} \frac{2x - b}{b} \quad \text{.....................(7)}$$

in which, the origin is at the heel; x is the abscissa; and b is the base width. The curve of Equation (7) is shown in *DA*, Fig. 9, indicating a steep hydraulic gradient near heel and toe (in fact, tangent to the vertical and, therefore, infinite immediately at the heel and toe), and a corresponding gentle intermediate gradient, dropping at the middle of the base to a value of $\frac{2}{\pi}$, or about two-thirds of the average gradient.

---

[12] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Mass. Inst. Tech., Cambridge, Mass.

[13] *Loc. cit.*, Equation (12).

This curve is of basic importance not only with reference to dams on sand, but apparently in the case of dams on rock foundations. Its general shape is corroborated by field observations of existing dams reviewed by Houk[14]. It should be evident that the curve would take this general shape instead of being straight because of the tendency of the water to follow the shortest path and thus crowd close to both heel and toe, increasing the velocity and, therefore, the rate of loss of head at these places. This curve will appear so frequently that the term, *S*-curve (Equation (7)), will be used for future reference.

It should be noted that Weaver's analysis is based on sand of indefinite depth, whereas in all practical cases the depth would be finite. To test the magnitude of the error thus introduced, the writer first attempted to verify Equation (7) by his analogy tray, using 20 in. for the base of the dam. The resulting experimental points, shown in Fig. 9, coincided as accurately with the *S*-curve as they could be platted. This proved both the amazing accuracy and consistency that can be realized by this experiment, and also that the limited size of the analogy tray would not vitiate the accuracy of other deductions for which no mathematical check is available. Most of the flow evidently follows closely adjacent to the structure and, within measurable accuracy, ignores the size of tray for a structure not more than 20 in. in base width.

The writer has extended Weaver's analysis mathematically to determine the upward escape gradient in the stream bed below the dam, which is a matter equally as important as base pressure. If the origin is transferred to the center of the base (see Fig. 9), and x is made equal to $\frac{B}{2}$, which is greater than $\frac{b}{2}$, and y remains a variable, Weaver's v being the internal pressure head in the material, then $\frac{dv}{dy}$ is the vertical component of the gradient, or is $\frac{h}{l}$ along a vertical line at the point where $x = \frac{B}{2}$. This gradient is found to increase toward the surface especially close to the dam, and at the surface, it becomes:

$$\frac{h}{l} = \frac{2\,H}{\pi\, b \sqrt{\frac{B^2}{b^2} - 1}} \qquad \text{.........................(8)}$$

in which, $B = 2\,x$; $\frac{h}{l}$ = the vertical escape gradient at the surface of the sand; H = head on the structure; and b = base width of dam.

---

[14] *Civil Engineering*, Vol. 2, No. 9, September, 1932, p. 578.

In Fig. 9 a curve of escape gradient is shown at the sand surface computed from Equation (8) for the condition of base width of twice the head, or $b = 2H$. The vertical scale of the escape gradient curve is in terms of $\frac{H}{b}$, or the mean hydraulic gradient along the base, the reciprocal of Bligh's percolation coefficient. The escape gradient at the immediate toe is infinite, indicating inevitable instability of sand at this point. The surface gradient decreases rapidly from infinite to a value of $\frac{H}{b}$ at a distance down stream from the toe of about $0.1b$. Because of the excessive escape gradient at the immediate toe as well as for obvious practical reasons, such as rain erosion, the base of a dam on soft material should never be flush with the foundation surface at the toe. A depressed toe or toe cut-off is required for all dams on sand.

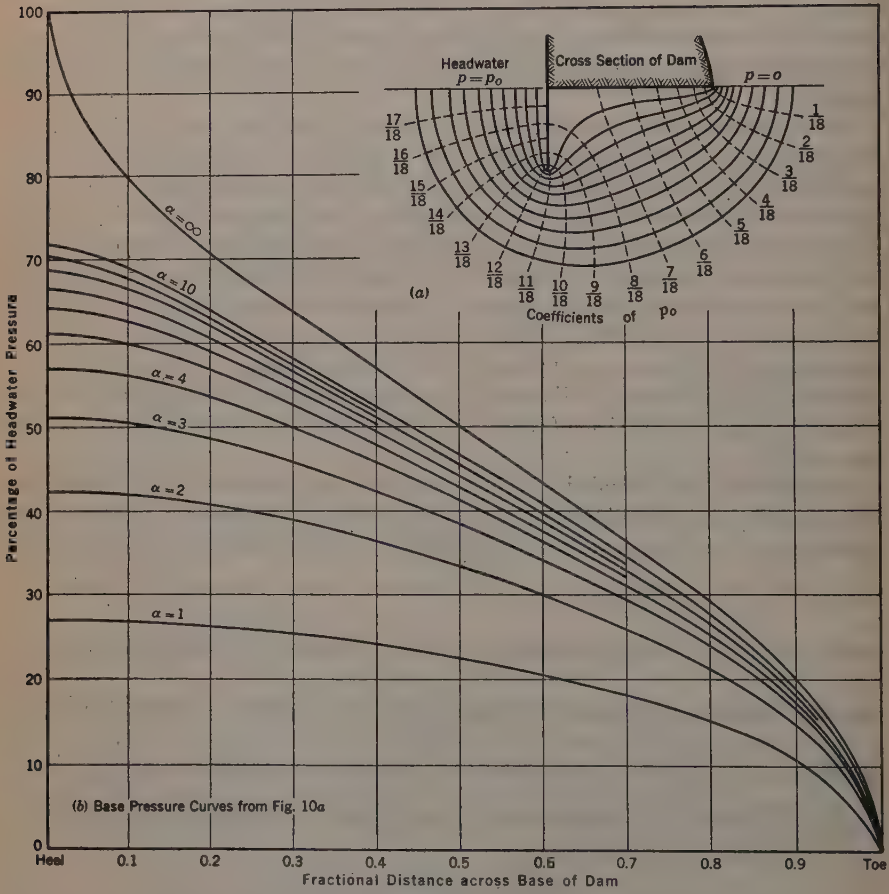

FIG. 10.—FLOW NET FOR HEEL CUT-OFF BUT NO TOE CUT-OFF.

Dam with Heel Sheeting but No Toe Sheeting.—For the case of heel sheeting, Weaver has determined mathematically, the curve of uplift pressure for various ratios of base width of dam to depth of sheeting. The lines of flow and equal pressure lines (flow net) as determined mathematically by him are illustrated in Fig. 10(*a*)[15], and agree with the flow net determined by H. M. Hill, Assoc. M. Am. Soc. C. E., between two closely spaced glass plates[16].

It will be noted that the pressure drop is nearly uniform along the up-stream face of the sheeting; and that it is very rapid due to crowding of flow lines around the tip of the sheeting, and again at the toe of the dam. The distributions of uplift pressure across the base of a dam are plotted in Fig. 10(*b*)[17] in percentages of head-water pressure, for various ratios, a, of width of base of dam to depth of sheet-piling at the heel. All these pressure curves become tangent to the vertical at the toe, which again indicates an infinite escape gradient and unstable material in close proximity to the toe as in the case of a dam without sheeting. This again should rule out this type of structure in which the base is flush with the foundation surface at the toe thus necessitating a depressed toe or toe cut-off.

Dam with Toe Sheeting but with Heel Apron in Lieu of Heel Sheeting.—This problem is mathematically and graphically the same as that of the last case except with the reversal of the direction of flow, Fig. 10(*a*). There would then be a rapid loss of pressure at the heel where the water enters the sand (which could do no harm), and a gradual and almost uniform rate of pressure drop as the water rises to escape on the down-stream side of the toe cut-off as shown by Fig. 10(*a*), if reversed.

The equation for pressure down the up-stream face of heel sheeting as derived by Weaver[18], by proper changes of sign for reversal of flow, becomes the equation for pressure at the depth, y, below the surface and along the lower face of the toe sheeting of the depth, d, as follows:

$$h = \frac{H}{\pi} \cos^{-1} \left[\frac{\lambda - 1 + \sqrt{1 - \frac{y^2}{d^2}}}{\lambda} \right] \quad \text{.................} (9)$$

in which, $\lambda = \dfrac{1 + \sqrt{1 + \frac{b^2}{d^2}}}{2}$; and $R = \dfrac{d^2 + x^2 - y^2}{d^2}$.

Regardless of its apparent complexity, Equation (9) is essentially straight when platted, except the bottom quarter near the tip of the sheeting.

---

[15] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Fig. 8.

[16] "Seepage Through Foundations and Embankments Studied by Glass Models", by H. M. Hill, *Civil Engineering*, January, 1934, p. 32.

[17] *Journal of Mathematics and Physics*, Vol. XI, No. 2, June, 1932, Fig. 5.

[18] *Loc. cit.*, Equation (27), p. 132.

TABLE 3.—COMPARISON OF ESCAPE GRADIENT, G

Ratio of base width to depth of sheet-piles, $\frac{d}{b} = a$	ESCAPE GRADIENTS IN TERMS OF THE RATIOS OF:			
	Head to depth of sheeting	Head to base width of dam	Head to creep distance	Head to short path of seepage
(*a*) DAM WITH TOE SHEETING AND HEEL APRON (SEE FIG. 11)				
1	$0.32 \frac{H}{d}$	$0.32 \frac{H}{b}$	$0.96 \frac{H}{L_c}$	$0.773 \frac{H}{L_p}$
2	$0.28 \frac{H}{d}$	$0.56 \frac{H}{b}$	$1.12 \frac{H}{L_c}$	$0.908 \frac{H}{L_p}$
3	$0.24 \frac{H}{d}$	$0.72 \frac{H}{b}$	$1.2 \frac{H}{L_c}$	$1.00 \frac{H}{L_p}$
4	$0.21 \frac{H}{d}$	$0.84 \frac{H}{b}$	$1.26 \frac{H}{L_c}$	$1.076 \frac{H}{L_p}$
5	$0.19 \frac{H}{d}$	$0.95 \frac{H}{b}$	$1.33 \frac{H}{L_c}$	$1.16 \frac{H}{L_p}$
6	$0.17 \frac{H}{d}$	$1.02 \frac{H}{b}$	$1.36 \frac{H}{L_c}$	$1.20 \frac{H}{L_p}$
8	$0.14 \frac{H}{d}$	$1.12 \frac{H}{b}$	$1.4 \frac{H}{L_c}$	$1.27 \frac{H}{L_p}$
10	$0.12 \frac{H}{d}$	$1.20 \frac{H}{b}$	$1.44 \frac{H}{L_c}$	$1.326 \frac{H}{L_p}$
(*b*) DAM WITH BOTH HEEL AND TOE SHEETING (SEE FIG. 12)				
1.5	$0.26 \frac{H}{d}$	$0.39 \frac{H}{b}$	$0.91 \frac{H}{L_c}$	$1.43 \frac{H}{L_p}$
2	$0.25 \frac{H}{d}$	$0.50 \frac{H}{b}$	$1.00 \frac{H}{L_c}$	$1.50 \frac{H}{L_p}$
2.5	$0.24 \frac{H}{d}$	$0.60 \frac{H}{b}$	$1.08 \frac{H}{L_c}$	$1.56 \frac{H}{L_p}$
5	$0.185 \frac{H}{d}$	$0.925 \frac{H}{b}$	$1.30 \frac{H}{L_c}$	$1.66 \frac{H}{L_p}$
(*c*) DAM WITH BASE SET 4½ INCHES INTO SAND (SEE FIG. 13)				
4.7	$0.16 \frac{H}{d}$	$0.75 \frac{H}{b}$	$1.05 \frac{H}{L_c}$	$1.05 \frac{H}{L_p}$

In Fig. 11 is given a set of diagrams indicating the base pressure in percentage of head from Fig. 10(*b*); and the writer's computation, from Equation (9), of pressure along the down-stream side of the toe sheeting both in percentage of head at indicated percentages of the depth of the sheeting. To facilitate the platting and comparison of curves the length of sheeting and base curves are platted as constant which is equivalent to foreshortening the horizontal scale of the base of the dam. The small inset diagrams indicate the true proportions. Only one of these diagrams was checked in the analogy tray, that of $b = 5d$. The platted points indicate an accurate agreement with the curves computed by formulas. In Table 3, the escape gradient is expressed as a function of $\frac{H}{d}$, $\frac{H}{b}$, $\frac{H}{L_p}$, and $\frac{H}{L_c}$, in which, L_p and L_c denote "short path" and "creep distance", respectively.

Dam with Both Heel and Toe Sheeting.—This case has not been analyzed mathematically by Weaver, and the writer has been unable to do so.

The close agreement of the electric analogy with the mathematical equations in the previous cases supports the belief that reliance can be placed on the former method. The writer has investigated four cases in his analogy tray the results of which are platted in Fig. 12. The first one used a 4-in. depth of sheeting on a 20-in. base width, all the others an 8-in. sheeting on base widths of 20, 16, and 12 in.

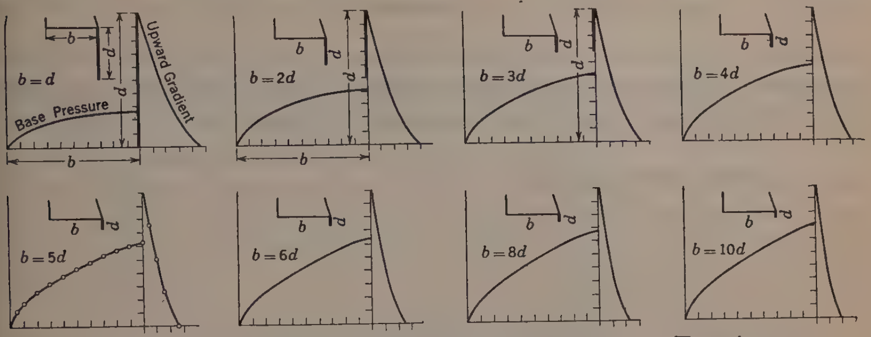

FIG. 11.—BASE PRESSURES, DAM WITH TOE SHEETING, AND HEEL APRON (SEE TABLE 3(*a*)).

Attention is called to the almost negligible difference in escape gradient for the three dams using 8-in. sheeting even with the large difference in base width (α = 1.5, 2, and 2.5 in Table 3(*b*)). This results, of course, from the small pressure drop under the base of the dam from the heel to the toe sheeting. The drop in pressure is much greater with the 4-in. sheeting as well as in the case of no heel sheeting. To assist in arriving at an explanation of this fact, the gap across the tips of the 4-in. sheeting was closed by a $\frac{1}{4}$-in. strip, thus simulating the condition of a flat base set a distance of $4\frac{1}{4}$ in. into the sand without heel or toe sheeting. The result-

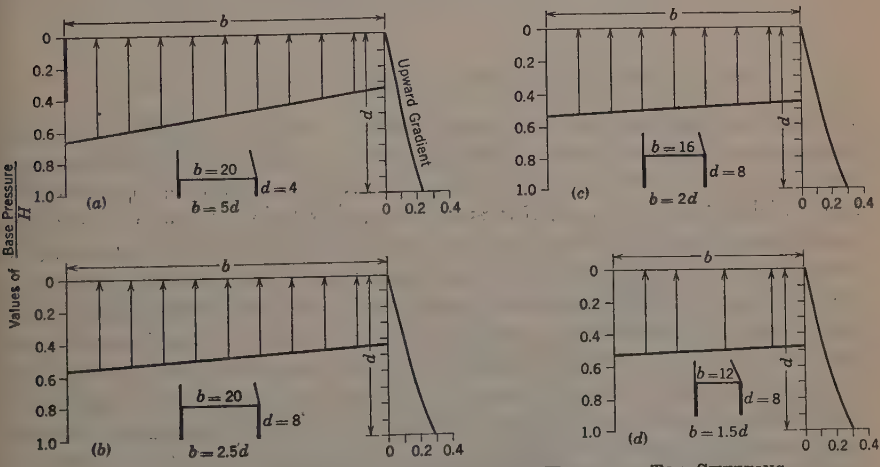

FIG. 12.—BASE PRESSURES, DAM WITH BOTH HEEL AND TOE SHEETING (SEE TABLE 3(*b*)).

ing loss of head along the base was much greater than with sheeting only (see Fig. 13) which served to reduce toe escape gradient from $0.925 \frac{H}{b}$ to $0.75 \frac{H}{b}$. These gradients reduce, respectively, to $1.66 \frac{H}{L_c}$ and $1.05 \frac{H}{L_c}$ (see Table 3(*c*)). Thus, for equal flotation safety, a Bligh percolation coefficient 1.58 times greater must be used for sheeting at the heel and the toe than for a base depressed equally into the sand. The explanation is apparently to

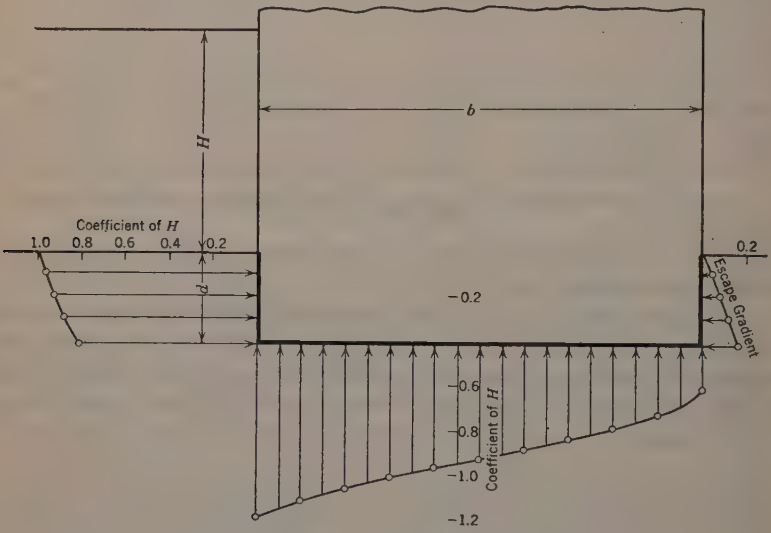

Fig. 13.—Base of Dam Set 4¼ Inches into Sand ($b = 4.7\,d$).

be found in the fact that the water flowing under the dam tends to concentrate as much as possible along the shortest path. With the depressed base, this flow is restricted by the structure from diffusing upward and, hence, follows largely a narrow strip along the base. With only sheeting to this same depth, the flow lines can spread both upward and downward between the lines of sheeting, with resulting loss of head slightly more than one-half as great. With the 8-in. sheeting the spread of the flow lines upward and downward is still more nearly equalized. On this theory it would seem that the minimum gradient between sheeting lines would be realized with sheeting extending nearly half-way to rock bottom, giving equal opportunity for diffusion upward and downward. Likewise, it would be expected (although not confirmed) that stratification would create more rapid loss of head by encouraging concentration rather than diffusion of flow.

Single Diaphragm of Sheet-Piling.—Especially for purposes of construction, but occasionally in permanent structures, a single diaphragm of sheet-piling serves as a dam unaided by additional base width. This case yields readily to mathematical analysis by making the base width, $b = 0$, in Equation (9), thus resulting in:

$$h = \frac{H}{\pi} \sin^{-1} \frac{y}{d} \qquad (10)$$

or,

$$\frac{dh}{dy} = \frac{H}{\pi\, d \sqrt{1 - \frac{y^2}{d^2}}} \qquad (11)$$

in which, h is the static head in the sand along the face of the sheeting of penetration, d, at a depth, y, below the sand surface, the sand being at the same elevation on both sides.

Fig. 14 shows the curve of Equation (10) platted to scale, and the values observed in the writer's analogy tray for sheeting 4 in., 8 in., and 12 in. deep.

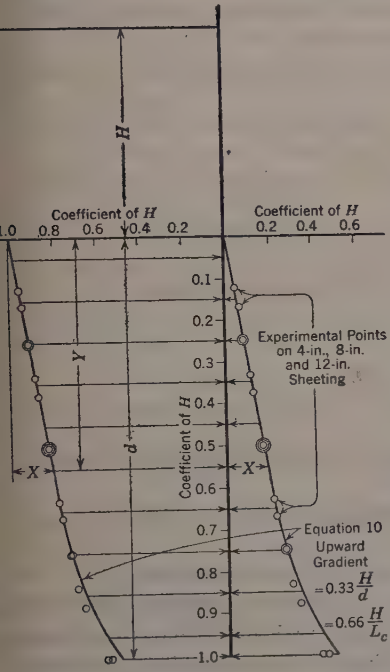

FIG. 14.—DAM WITH SINGLE ROW OF SHEET-PILING

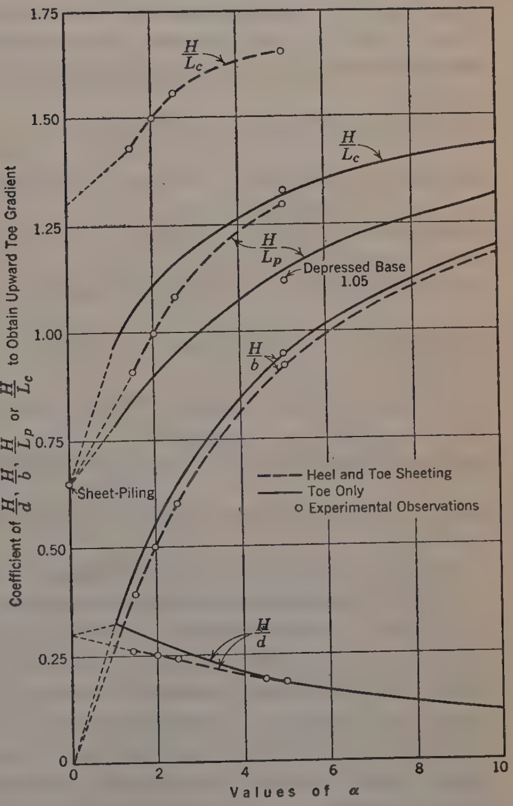

FIG. 15.—MATHEMATICAL AND EXPERIMENTAL DETERMINATION OF ESCAPE GRADIENTS.

The verification is close within the possible limits of accuracy in platting, except near the bottoms of the curves. This is explainable by the fact that the sheeting used in the analogy tray was $\frac{1}{4}$ in. thick and the bottom observations were at the lower corners, slightly more than $\frac{1}{4}$ in. apart, instead of on the bottom of the sheeting strip at the center line. The loss of head around the point of sheeting is rapid. The escape gradient up the face of the sheeting is:

$$G = 0.66 \frac{H}{2d} \qquad (12)$$

or two-thirds of that which would be expected from uniform loss of head along the creep distance. The case of unequal sand levels on opposite sides of the sheeting has not as yet been investigated.

SIGNIFICANCE OF "LINE-OF-CREEP" AND "SHORT-PATH" THEORIES

The logic of the application of Bligh's coefficients to the long path or line of creep, or to the short-path distance can now be investigated. The results of mathematical and experimental determination of escape gradients from Figs. 11 and 12 for the cases of toe sheeting only, and for both toe and heel sheeting of equal length, are assembled in curve form in Fig. 15 which is platted against a horizontal scale of $\alpha = \frac{b}{d}$ for comparison. In each curve the flotation gradient at the toe is obtained by application of the coefficient at the left to $\frac{H}{d}$, $\frac{H}{b}$, $\frac{H}{L_p}$, or $\frac{H}{L_c}$, all arriving at the same result. These curves are dotted to the points where they should intersect the line, $\alpha = 0$, equivalent to a single row of sheeting at 0.33 for $\frac{H}{d}$; 0 for $\frac{H}{b}$; 0.66 for $\frac{H}{L_p}$ and $\frac{H}{L_c}$, for one cut-off, and 1.32 for $\frac{H}{L_c}$ for a double cut-off. The fact that these curves do not tend exactly toward the foregoing values for $\frac{b}{d} = 0$, is believed to result from the thickness of the model sheeting, the same cause explained under "Single Diaphragm of Sheet-Piling".

If the factor of safety against percolation could be represented correctly by applying a coefficient to any of the foregoing ratios, the curve of such expression in Fig. 15 would be a horizontal line. This is far from the case except for sheet-piling, which is a single point and for which the line-of-creep theory is, therefore, correct in principle. The other curves vary through a wide range of coefficients indicating that none of the heretofore accepted bases applies correctly to a solution of the problem. The writer has been unable to devise any method of weighting the several factors involved which would yield a nearly horizontal line in Fig. 15. This is not surprising when the complexity of the problem is observed from Equation (9).

The required increase of coefficients with increasing ratios of $\frac{b}{d}$ (Fig. 15), indicates that a constant coefficient as by Bligh would provide a smaller factor of safety for increasing values of $\frac{b}{d}$. Bligh's factors thus tend to yield a higher factor of safety for modern types of dams, both because of their smaller values of $\frac{b}{d}$ and because of the monolithic types of construction as compared with the older types, which often consisted of dry rock or rubble fills between concrete walls, and shallow cut-offs, if any. In fact, Bligh's coefficients were developed for some dams in which seepage could escape progressively along the down-stream slope instead of being confined until reaching the toe.

Consistency of Bligh Coefficients.—It is desirable to inquire into the consistency with which these coefficients can be applied to different types of foundations. Thus, assume $\frac{b}{d} = 5$, with an escape gradient of 0.25, and determine what value of the Bligh coefficient, C, will be required to assure this gradient.

For a single line of sheeting, (Fig. 15), $0.25 = 0.66 \frac{H}{L_c}$; or,

$$C = \frac{L_c}{H} = \frac{0.66}{0.25} \dots\dots\dots\dots = 2.64$$

For other types the values of C are in the same ratios as the ordinates of the $\frac{H}{L_c}$ curves in Fig. 15 for $\frac{b}{d} = 5$; or,

$$C\text{, for depressed base (Fig. 13)} = \frac{1.05}{0.66} \times 2.64 \dots\dots = 4.17$$

$$C\text{, for heel apron and toe sheeting} = \frac{1.33}{0.66} \times 2.64 \dots\dots = 5.3$$

$$C\text{, for heel and toe sheeting} = \frac{1.66}{0.66} \times 2.64 \dots\dots = 6.6$$

Thus, for dams of equal safety from toe flotation, C varies from 2.64 to 6.6. This fact indicates that these coefficients, based upon creep distance, are not representative of true resistance; they are even less representative from a practical rather than a theoretical standpoint when the effect of stratification and possible roofing is considered. It would seem self-evident that the art of design against percolation admits of improvement.

The high resistance furnished by sheeting will also serve to explain the previously surprising performance record of the Prairie du Sac Dam, in Wisconsin, which was dependent for several years on 50-ft. heel sheeting alone for a head of about 34 ft on medium fine sand. This represents a percolation coefficient of about 3, which is generally considered too small for safety; and yet, in reality, it represents in this case a factor of safety of at least 4, perhaps 5, against flotation (knowledge of porosity of the material is not available).

Observation of Fig. 16 for 8-in. heel and toe sheeting reveals that ther is little dissipation of head on the down-stream side of the heel sheeting, o the up-stream side of the toe sheeting, and in the horizontal run between th sheeting. The result is that combined heel and toe sheeting acts muc like a single line with little gain in effectiveness because the dissipation of head, largely occurs on the upper side of the heel sheeting, on the lower side of the toe sheeting, and around the tips of the sheeting.

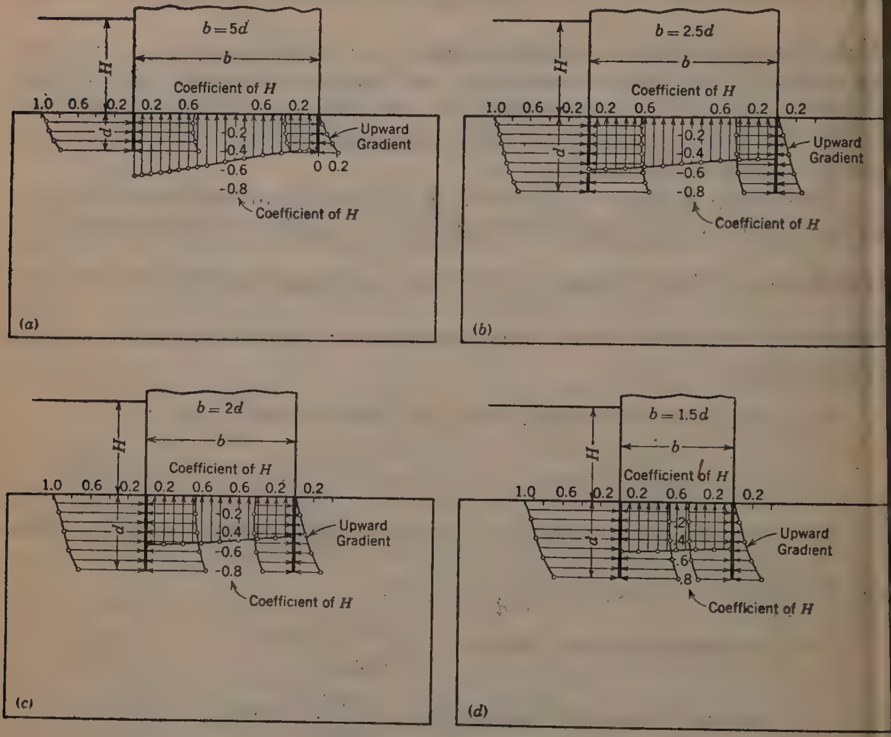

FIG. 16.—HEEL AND TOE CUT-OFFS.

PROPOSED RATIONAL APPROACH

The writer proposes, therefore, the gradual abandonment of Bligh's coefficients as fast as data regarding existing dams can be converted into the form of a safety factor or coefficient, f, to be applied to the critical flotation gradient, Equation (6), to arrive at a safe escape gradient, G, or,

$$G = \frac{F_c}{f} = \frac{1}{f}(1 - P)(s - 1) \quad \text{.................} \quad (13)$$

The value of f is to be chosen with reference to the geological details, uniformity of the sand, stratified condition, and extent of knowledge of the sub-surface conditions. This factor, f, will then be dependent entirely upon judgment, empirical data, and field knowledge of foundation conditions constituting a true coefficient, whereas Bligh's coefficients involve also a part of the mathematical principles of the problem, which should be segregated. Once the basic equations and curves are available, it can be made as easy to use correct mathematical principles with modifying coefficients or factors of safety (which are truly dependent only on the non-mathematical and undiscernible elements of the problem), as to use Bligh's coefficients. Considerable research by the analogy method will be necessary to produce designing curves for all types of foundations, such as the double line of sheeting of unequal length, three or more lines, of sheeting, sloping surfaces, etc. In general, such research must be left to be conducted for the specific problem at hand. However, enough has been done herein to furnish a basis for the design of three types: (*a*) Heel and toe sheeting of equal length; (*b*) heel apron combined with toe sheeting; and (*c*) a single line of sheeting acting by itself.

Application of Escape Gradient to Design Problems.—The critical flotation gradient for average quartz sand with $s = 2.65$ for sands of various porosity as computed from Equation (6), are:

Porosity	Critical Flotation Gradient, F_c
0.30	1.15
0.35	1.07
0.40	0.99
0.45	0.91
0.50	0.825

It will be noted that the foregoing critical or flotation gradients, F_c, are approximately unity. Their reciprocals or Bligh's coefficients, therefore, would also approximate unity without a factor of safety if the average gradient along the line of creep were equal to the upward component at the point of escape. Fig. 15 furnishes a practical solution of design problems for foundation types to which it applies. Thus, if G is the escape gradient; m, the coefficient of $\frac{H}{b}$; and n, the coefficient of $\frac{H}{d}$, as ordinates at the left of Fig. 15, then, for any common value of $\frac{b}{d}$, $G = \frac{m\,H}{b} = \frac{n\,H}{d}$; or,

$$\frac{H}{G} = \frac{b}{m} = \frac{d}{n} \qquad (14)$$

The head, H, is always known; the allowable value of G is determined from the physical properties of the sand modified by a factor of safety, Equation (13), and either b or d must be assumed. Thus, assume a double-sheeted dam with a head, H, of 30 ft and a quartz sand of about 40% voids, with a critical flotation gradient of 1.0 and a factor of safety of 4 yielding a working escape gradient of 0.25, then, $\frac{H}{G} = \frac{30}{0.25} = 120 = \frac{b}{m} = \frac{d}{n}$. If $b = 90$, then $m = 0.75$, $\frac{b}{d}$ from Fig. 15 $= 3.5$, and, hence, $d = \frac{90}{3.5} = 26$ ft.

DRAINAGE

The principle of drainage can be applied both to reduce upward pressur and to increase the factor of safety against flotation at the toe. Drainag is dangerous in the manner in which it is often carelessly installed (such as fragile tile in ungraded gravel trenches), and may permit the escape o

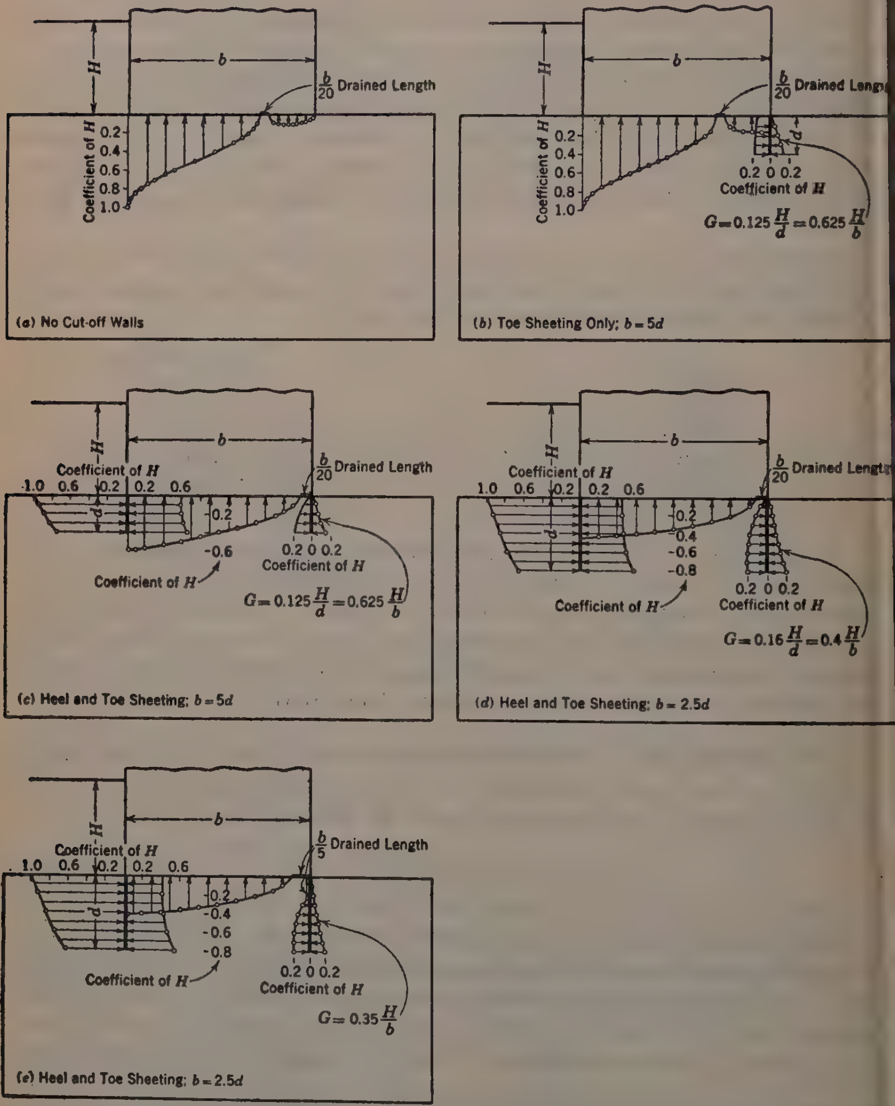

FIG. 17.—EFFECT OF DRAINAGE ON A DAM WITH FLUSH FOUNDATION.

material from beneath the structure. To be safe, drainage must follow the principle of an inverted sanitary water filter. There must be at least two layers of sand and one of gravel screened to such sizes that the material of one layer cannot penetrate the pores of the next layer above. Perforated plates or similar venting means must permit free escape of water through the structure.

The effect of drainage has been investigated by the writer in the analogy tray. Thus, in Fig. 17(*a*), is shown the result of draining the 15th-in. space of a 20-in. foundation without heel or toe sheeting. The uplift pressure curve to the point of drainage is the basic *S*-curve, Fig. 9, with the base reduced to three-fourths length. Beyond the point of drainage a small uplift pressure is resumed with a curve tangent to the vertical at the toe. This results in an infinite upward gradient at the immediate toe as in the basic *S*-curve. In Fig. 17(*b*) the same conditions are repeated except with a length of toe sheeting equal to $\frac{b}{5}$. The sheeting has no effect up stream from the drainage and serves principally to change the shape of the pressure curve at the toe to give a finite escape gradient. Comparing this with the same design ($b = 5\,d$ in Fig. 11), except without drainage, the reduction in escape gradient is found to be from $0.19\,\frac{H}{d}$ to $0.125\,\frac{H}{d}$, or a decrease of 34%, which is not as great as might be expected.

In Fig. 17(*c*) is shown heel and toe sheeting ($b = 5d$), with drainage 1 in. long immediately ahead of the toe sheeting; the resulting upward toe gradient, $0.125\,\frac{H}{d}$, agrees with that of the last case (see Table 3(*e*) and Table 3(*f*)). Another test was made of a dam with an 8-in. heel and toe sheeting, in which $b = 2.5\,d$, with a 1-in. drainage strip as in Fig. 17(*c*). The upward toe gradient is $0.4\,\frac{H}{b}$ as compared with $0.6\,\frac{H}{b}$ without drainage (compare Table 3(*g*) and Table 3(*b*). This test was repeated with a 2-in. drainage strip immediately above the sheeting which resulted in a gradient of $0.35\,\frac{H}{b}$. This was again repeated with drainage along the upper 2 in. of the toe sheeting as well as 2 in. of the adjacent base width and with the upward toe gradient of $0.35\,\frac{H}{b}$ unchanged from the last one.

As in the case of earth dams there are two schools of thought with reference to drainage, those generally for, and those generally against, it. The writer believes that it provides a useful factor of safety especially in uncertain ground. He believes in the principle of imposing effective resistance to seepage through such a distance as is required until the quantity of leakage is sufficiently reduced and then providing ample artificial, permanent, and well-constructed drainage works which will relieve as much as possible through a controlled route where its escape will be easier than through the material and where it can do no harm. Only the escape of seepage through the material is hazardous and it is better to provide a route through which it can escape more freely that through the material itself. It must be kept in mind that seepage will follow all routes of escape in inverse proportion to their resistances; and drainage, therefore, becomes a safety factor, not an elimination of toe-flotation possibilities.

Thus, any pervious stratum or lense that may conduct execessive pressure beneath a dam, possibly to burst upward into tail-water, should be intercepted or blanketed near the heel where possible; and then near the toe it should be drained as freely as possible. Water will not burst upward through the sand from internal pressure if there is an easier route of escape than by lifting the sand. This line of escape may be created by well-points, screened driven wells, or intercepting trench if necessary to gain capacity, or (if local) by a dug well. Drainage wells can be connected by a header or filter gallery under or in the apron or toe of the dam and carried to a pump sump if it is desirable to hold the drained level below tail-water. This need might readily arise to protect an apron against uplift ahead of the hydraulic jump. Water would then drain toward the filter gallery from both head-water and tail-water. A complete line of drainage along the down-stream face of the toe sheeting could be used to eliminate most of the upflow through the material itself in an unstable situation requiring such precaution. Drainage is a useful tool for the designer and is capable of a great variety of useful applications.

HORIZONTAL POROUS STRATIFICATION

No Cut-Offs.—It was next attempted to gain some idea of the influence of horizontal pervious strata. To do so a 22-gauge, a ¼-in. strip and, alternately, a 1-in. strip of copper were submerged in the tray, successively, at the four positions shown in Fig. 18(*a*). The conductivity of the copper so greatly exceeded the salt solution as to be equivalent to a free-flow, no-resistance, stratum. Practically no difference was found in the ohmic resistance between forebay and tail-race as between the 1-in. and ¼-in. strip. Therefore, the latter was used exclusively afterward. Fig. 18(*a*) shows the variation in resistance of the bath with the varying positions of the copper strip.

In Fig. 18(*b*) the uplift pressure curves under the base are platted for a dam with no cut-offs: (*A*) On homogeneous material, and with no copper strip (the typical *S*-curve of Fig. 9); (*B*) with the copper strip, $\frac{b}{2}$, or 10 in. below the base; and (*C*) with the strip, $\frac{b}{4}$, or 5 in. below the surface. Curve (*D*) applies to the last case (*C*), but with a wire jumper from the copper strip to the terminal at the toe to simulate the result of freely draining the pervious stratum to the tail-race; the form of curve is entirely changed and an infinite toe gradient is replaced with zero gradient.

With Cut-Offs.—The first experiment, Fig. 18(*c*), finds the copper strip 2 in. below the tips of the 8-in. sheeting and not intercepted by it, which

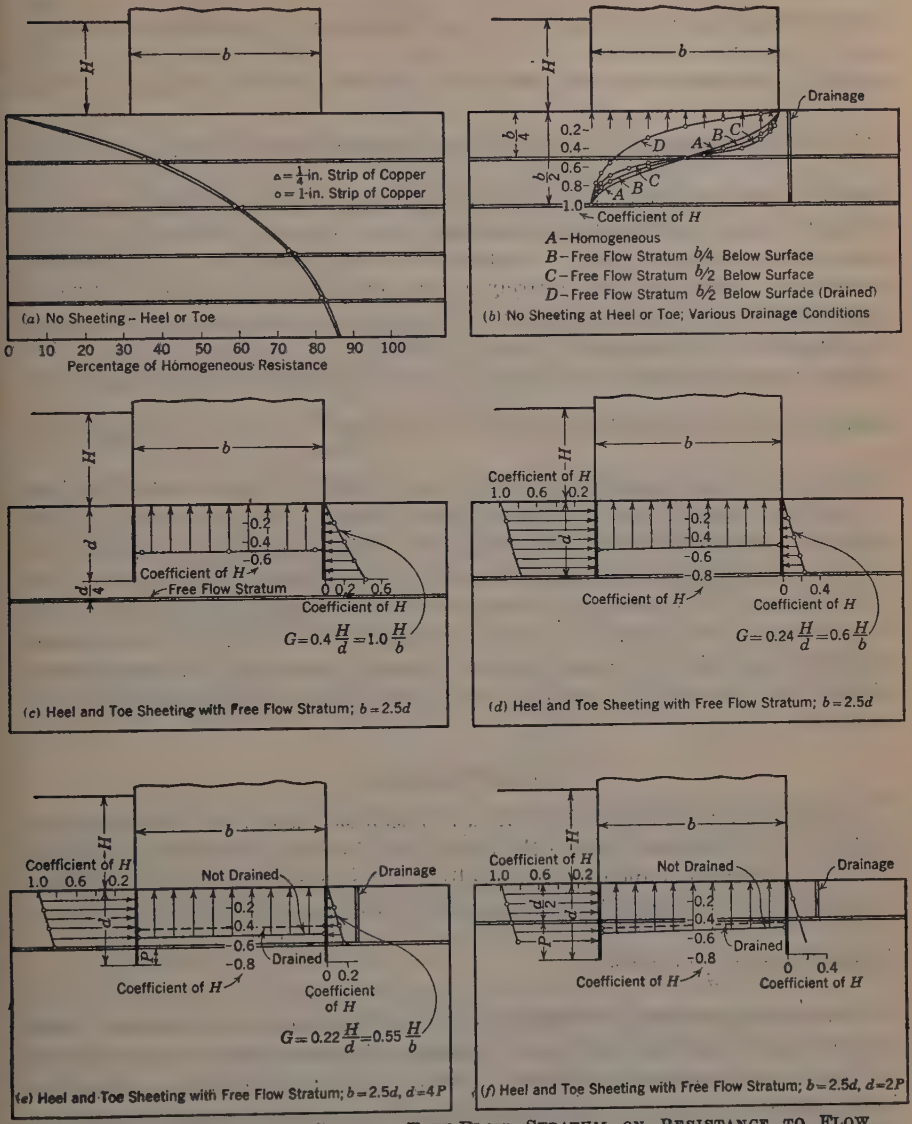
H
b
△ = $\frac{1}{4}$-in. Strip of Copper
○ = 1-in. Strip of Copper
(*a*) No Sheeting – Heel or Toe
0 10 20 30 40 50 60 70 80 90 100
Percentage of Homogeneous Resistance
Drainage
b/4
b/2
0.2
0.4
0.6
0.8
1.0
D
A
B
C
Coefficient of H
A – Homogeneous
B – Free Flow Stratum b/4 Below Surface
C – Free Flow Stratum b/2 Below Surface
D – Free Flow Stratum b/2 Below Surface (Drained)
(*b*) No Sheeting at Heel or Toe; Various Drainage Conditions
d
d/4
Coefficient of H
Free Flow Stratum
0 0.2 0.6
$G = 0.4\frac{H}{d} = 1.0\frac{H}{b}$
(*c*) Heel and Toe Sheeting with Free Flow Stratum; b = 2.5d
1.0 0.6 0.2
0 0.4
$G = 0.24\frac{H}{d} = 0.6\frac{H}{b}$
(*d*) Heel and Toe Sheeting with Free Flow Stratum; b = 2.5d
Not Drained
Drained
P
0 0.2
Coefficient of H
$G = 0.22\frac{H}{d} = 0.55\frac{H}{b}$
(*e*) Heel and Toe Sheeting with Free Flow Stratum; b = 2.5d, d = 4P
d/2
(*f*) Heel and Toe Sheeting with Free Flow Stratum; b = 2.5d, d = 2P

FIG. 18.—EFFECT OF SUB-SURFACE FREE-FLOW STRATUM ON RESISTANCE TO FLOW.

results in an escape gradient, $0.4\,\frac{H}{d}$, or $1.0\,\frac{H}{b}$. The gradient is higher than the gradient of a single row of sheeting $\left(0.33\,\frac{H}{d}\right)$ because there is no resistance between sheeting and less than otherwise past the tips.

To simulate an intercepted stratum the $\frac{1}{4}$-in. copper strip was next cut in sections to fit between the sheeting and the tray walls. The case of open stratum barely penetrated is shown in Fig. 18(*d*). The gradient is $0.24 \frac{H}{d}$, or $0.6 \frac{H}{b}$, the same as in the case of no pervious stratum. An open stratum penetrated a distance, $\frac{d}{4}$ (Fig. 18(*e*)) gives a gradient of $0.22 \frac{H}{d} = 0.55 \frac{H}{b}$, which is even less than with no pervious stratum. With penetration of 4 in. (Fig. 18(*f*)), the gradient was the same as in the case of Fig. 18(*e*). This apparent anomaly can only be explained by the tendency of the free-flow stratum to carry the flow entirely to the cut-off without much diffusion, from whence it is concentrated at a high-current density around the tip of the cut-off with consequently rapid dissipation of pressure. In fact, two distant lines of independent sheeting with no resistance between them would have an upward toe gradient each of one-half $0.33 \frac{H}{d} = 0.165 \frac{H}{d}$. Apparently, this limit is being approached as the copper strip is raised toward the base of the dam, but will not be reached without wider spacing of sheeting. This apparent anomaly is significant in connection with roofing, and indicates that an open space under the foundation tends toward greater safety from toe flotation, assuming, of course, in all cases herein the integrity of the cut-offs. In the last two cases a further experiment was made by draining the open stratum below the toe with the result that no pressure existed on the down-stream side of the toe sheeting and that the uplift pressures under the base were reduced as shown.

OTHER COMMENTS

Uplift Under the Base.—Not much has been said about the important subject of uplift pressures because the curves are obvious. It is to be noted that in all symmetrical foundations the uplift at the center line is 0.50 H. With a double row of sheeting the variation of this pressure from heel to toe becomes less as $\frac{b}{d}$ decreases.

The uplift pressures obtained from the analogy are static pressures. They take no account of the force in the direction of flow exerted by the movement of the water through the material. Thus, in the drained bases, a considerable lifting effort must necessarily be exerted as the water rises to the drains, but will not increase the base pressure unless the upward gradient exceeds the critical value (see Equation (7)).

Sheeting Cut-Offs.—Experience seems almost universally to confirm some degree of uncertainty of effective sheet-piling cut-offs, except in favorable material, due to the possible opening of interlocking joints in driving. Unfortunately, the interlocking sections with the greatest jaw strength are difficult to drive in sand and necessitate compacting the sand intensely to force it out of the interlock ahead of the sheeting being driven. Boulders are a source of trouble, sometimes causing splitting of piles or separation of interlock without the knowledge of the operator. Much depends on the skill and experience of the operator. The most experienced can perhaps detect when all is not well with the sheeting, but there always remains a lurking uncertainty unless, as was done in a recent project, each sheet-pile was driven, pulled to determine its condition, and driven again. This, of course, is not practicable. The driving of sheet-piles in sand and gravel is a tedious slow operation and delays the work on the superstructure until this is complete. There are often boulder beds through which the sheeting cannot be driven successfully.

Trenched Concrete Cut-Offs.—The provision of a concrete cut-off in an excavated trench is sometimes resorted to in order to avoid the uncertainty of sheeting, and this is probably to be preferred. However, it is usually more expensive, causes still greater delay in initiating work on the superstructure, and has some (although less) element of uncertainty due to possible cracking apart from the main structure.

Abutments.—Too many dams are designed from a cross-section which ignores abutment conditions (in La Fruta Dam, Corpus Christi, Tex., for example). The abutment seepage problems are similar to those of the dam tipped on end, but are often more serious and difficult than those encountered under the dam itself. In the latter, for example, the seepage under a 50-ft length of dam has a 50-ft. length of toe under which to escape; but the seepage around the abutment of a dam, 50 ft high, all tends to concentrate at or near the tail-water line. This requires special attention to provide both ample lateral cut-offs and drainage.

RESEARCH BY MODEL TESTING

The studies made by the writer and reported herein have scarcely "scratched the surface" of needed and possible research as to the almost infinite variations of types of foundations used for dams on sand. Only a few of the elementary types justify general research to establish general principles. It is recommended, however, that for specific design problems, the alternate plans under consideration, and their fundamental hydraulic differences, be investigated by the analogy method as a useful guide to judgment. It is very accurate in principle and application, thus excluding the necessity of mathematics, which becomes too involved except for the simplest cases.

It is also believed that the analogy principle can be applied to three-dimensional models, thus permitting investigation of such problems as the influence of diagonal flow and relative concentration, if any, in the thread of the underflow stream (deepest channel), abutment conditions, etc. For this purpose a model of the entire structure would be made as a water-tight boat of insulating material, such as bakelite, with the proposed abutements correctly reproduced. Metallic screws would penetrate the boat shell from the inside at frequent intervals upon which voltage readings could be taken.

Sheet-metal surfaces would simulate the sand surface in the river bed up stream and down stream of the dam; and the voltage would be applied between them.

Existing Dams.—It is suggested that some properly equipped research organization should undertake the determination of fundamental hydraulics by the analogy method on base-profile models of existing dams of importance. The aim should be to determine the theoretical pressure along the entire line of creep; the dissipation of head by various details of base profile, etc.; and the comparison of this with field data wherever records are available of actual pressures along the line of creep. The accumulation of such comparisons would be of inestimable value to designers, in arriving at the magnitude of departures to be expected from theoretical conditions, and the factors of safety to be used in design, effect of filter skin, etc.

SUMMARY AND CONCLUSIONS

1.—The problem of the safe conduct of seepage under a dam is analogous to that of the safe conduct of water over a spillway, in the sense that it seeks means to dissipate the head without disturbing or carrying away the foundation or stream-bed material.

2.—The dissipation of head is gradual through the foundation material from head-water to tail-water. The difference in head, as between two points, exerts a force against the intervening material and also causes a flow of pore water to occur. The reduction in pressure is the "cause"; force and seepage are sister 'effects", material as well as water tending to be moved by the reduction in internal hydrostatic pressure.

3.—Seepage and force both occur or act in the direction of maximum reduction of pressure; that is, in a direction normal to surfaces or lines of equal pressure. The force exerted on a volume of material is, therefore, in the direction of seepage and is equal to the difference in internal pressure or head acting on the approaching and receding faces of the volume as against a solid rather than a porous surface.

4.—Foundation material cannot move when it is confined, unless the entire foundation as a whole is unstable. If not collectively unstable, then no material can be carried away by seepage unless and until the material ahead of it, in the direction of seepage, is first removed, as otherwise it would have no place to go.

5.—Analysis indicates that, in general, material under a dam is in stable condition with forces having a safe component downward until the tail-water is approached, at which point the drop of internal pressure has a component upward, becoming vertical at the surface. The more rapid the upward reduction in pressure (that is, the higher the upward hydraulic gradient), the less becomes the effective weight of the material until at a critical value, when the escape gradient equals $\frac{h}{l} = (s - 1)(1 - P)$ (see Equation (6)), the material actually floats and may be carried away with resulting rapid crumbling from the toe backward under the dam, causing failure. The tendency of this phenomenon to localize at the point in the tail-water of easiest escape may cause local sand "boils" and may give the impression that the sand is moving simultaneously throughout the route from head-water, thus producing an effect known as "piping". Actually, it is not, as is evident from Fig. 5; the instability is a local toe phenomenon, best termed, toe "flotation"; the problem is to prevent flotation, and piping cannot occur. Whether or not the critical value is approached, there will always be some upflow at the toe and this will reduce the effective weight of the sand and make it more easily eroded by waves, current, or rain.

6.—There seems to be no logical basis for expressing safety against toe flotation in terms of "creep distance", "short path" of seepage, or any functions thereof.

7.—Safety from toe flotation is best promoted:

(*a*) By choosing a design with a depressed toe or toe cut-off and one which dissipates head rapidly along the early part of its route, leaving as little remaining head as possible to be lost during the upward flow into tail-water;
(*b*) By supplying a special fill down stream from the toe, an inverted filter in principle;
(*c*) By providing an inverted filter drain under the toe of the structure itself, ahead of the toe cut-off; and
(*d*) By means of wells, drainage galleries, etc., at the toe.

8.—Dissipation of head is proportionate to velocity, which, in turn, is dependent on the tendency of the flow to concentrate. Such concentration occurs, especially, around protruding corners of the structure and about the tips of cut-offs, as well as along the up-stream face of heel cut-offs and the down-stream face of toe cut-offs. A single line of sheeting is the most effective structure investigated per unit of creep distance for the dissipation of head.

9.—A depressed toe or toe cut-off is theoretically essential for any dam, in order to avoid infinite upward gradient at the immediate toe as well as to prevent undercutting by erosion of rain, current, and wind. A heel cut-off in homogeneous material of relatively great depth, is apparently little more effective as to toe escape gradient than a heel apron, and is more expensive.

10.—The subject of drainage permits of much greater elaboration and more varied use than it has ever received, and much can be accomplished by drainage to improve the present state of the art.

11.—Hereafter all dams on sand should be equipped with sounding wells, pressure-recording devices, and other means of observing constantly the pressure conditions along the entire line of creep.

12.—Mathematics and research by means of hydraulic electric analogy offer valuable information as to fundamental principles of dissipating head in homogeneous material and relative ability of different foundation types to promote the minimum upward hydraulic gradient at the toe. Although the heterogeneity of natural deposits and the difficulty of interpretation by test borings will always require the use of a factor of safety above theoretical requirements, the writer maintains that a study of the problem should start from a nucleus based upon the law of flow through homogeneous material, with modifications introduced in the analogy tray in each individual case based upon such local conditions as can be revealed. Field tests and experience should be grouped around such ideal assumption instead of about purely empirical and unscientific coefficients, such as those of Bligh to the end of eventually determining the order of magnitude of variations from the ideal by coefficients, or by factors of safety, much as in the case of weirs, etc.

13.—It is submitted that the proper ultimate method for designing dams on sand is to compare various proposed alternate types of foundation cross-section and abutment designs by the analogy method (simulating impervious and pervious strata as nearly as known). for uplift and upward toe gradient, amount of seepage, cost, etc.; and then to modify these comparisons based on field conditions at the site and empirical data which will have been accumulated in the meantime to aid in interpreting such conditions into their bearing upon the criteria of safety.

14.—The extent that "roofing", stratification, and other effects of non-homogeneity may modify the theoretical pressure profile in practice, should be observed in the analogy tray, and in actual structures and should be compared with the pressure profile that would exist in homogeneous material. It is proposed, therefore, that analogy models be made of all existing dam foundations for which foundation pressure records exist, in order to segregate the mathematical law from the effect of non-homogeneity of material, filter skin, roofing, etc.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SOME SOIL PRESSURE TESTS

Discussion

By H. de B. Parsons, M. Am. Soc. C. E.

H. de B. Parsons,[30] M. Am. Soc. C. E. (by letter)[30a].—This research was made to study the horizontal pressure of a back-fill against a bulkhead or retaining wall. The numerical values apparently reflected a movement of the bulkhead. The results demonstrated (1) that there was a reduction in pressure at early stages of irrigation; (2) that the repetitions of irrigation and drainage cause no permanent change in the pressures; and (3) that there is a difference between internal friction and internal resistance of soil grains under wetted conditions. The second result is akin to a quay wall against which a tide rises and falls.

While compiling the test results, the writer conceived the theory that atmospheric pressure may have caused the reduction of the soil pressure when the soil was saturated. This theory will be elaborated more fully hereafter.

In Mr. Meem's box of 1-ft cube, the forces set up by tightening the bolts do not correspond with the pressures discussed in the paper. Mr. Meem evidently refers to "arching" as affecting the horizontal pressures of the tests. The repeated tests gave uniform results, and had arching existed to any marked extent, the results would have shown greater variation. Mr. Meem's idea of an apparatus having a wide width would help to avoid doubt as to arching, although increasing the expense of a test. The writer did consider a testing apparatus having water or oil rams for measuring the pressures, but he abandoned the rams because they were not as delicate as the scales for recording small differences in pressures.

Mr. Meem is correct in stating that the plane of rupture is curved, but he is in error in describing the angle of repose as between the vertical and

---

Note.—The paper by H. de B. Parsons, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1934, by J. C. Meem, M. Am. Soc. C. E.; February, 1934, by Messrs. Eugene E. Halmos, and L. C. Wilcoxen; March, 1934, by Messrs. O. K. Froehlich, H. L. Thackwell, and Jacob Feld; April, 1934, by Messrs. T. Farrance Davey, D. P. Krynine, and Charles Terzaghi; and May, 1934, by Messrs. R. L. Vaughn and M. Hirschthal.

[30] Prof. Emeritus, Rensselaer Polytechnic Inst., Troy, N. Y.; Cons. Engr., New York, N. Y.

[30a] Received by the Secretary July 20, 1934.

the plane of repose. It is the angle between the horizontal and the plane of repose; and the planes of repose and of rupture are not identical.

Mr. Halmos was familiar with the manuscript of the paper, and concurred with the writer that minute possible movement of the bulkhead would not materially affect the results. Neither he nor the writer knew, at that time, the results of the experiments conducted at the Massachusetts Institute of Technology,[31] and no doubt Mr. Halmos would modify his opinion, in the light of these experiments, which showed that a slight movement of a bulkhead, or wall, decreased the active pressure.

Mr. Wilcoxen's analysis of the reduction in the pressure of the bulkhead, when the lower strata of the back-fill were immersed, is interesting; and his idea of capillarity may account for some soil phenomena. The writer believes that the irrigating water entered too fast for active capillary action to occur. Capillary water rises above gravity water by creeping between the particles due to some attractive force. Voids, left by the upward-moving capillary water, would be re-filled by water, and this water movement would be resisted by its surface tension and by friction of the water moving over the surfaces of the soil particles. This is a slow process, and capillarity requires time.

The unit weight of the sand in the test bin averaged more nearly the rodded weight of 111 lb than the rammed weight of 112.5 lb, as used by Mr. Wilcoxen. As the effective height of 6.906 ft was used, he should also have used the effective width of 4.75 ft instead of the full width of 4.926 ft from canvas to canvas.

It is of interest that Mr. Wilcoxen has referred to vacua as affecting the pressures in a mass of soil, and that Professor Terzaghi, in analyzing the Massachusetts experiments[32] has also referred to vacua in his "suction-head", as both these ideas are analogous to the writer's vacuum theory,[33] which conceives a change due to atmospheric pressure in a soil under certain conditions, as will be mentioned more fully elsewhere.

Dr. Froehlich's eight premises are not open to argument, except that the weight of the soils as recorded were close average values rather than approximate. The "rodded" weights, in Table 2, were considered to be closer to the average actual unit weights of the compacted materials in the testing bin, than the "rammed" unit weights, and the former were used by the writer. Dr. Froehlich considered that the bulkhead and bin walls were not rigid, and that movements, both outward and inward, took place during the tests. The side walls were strongly braced by waling-pieces and outside diagonal braces; they were always wet after the standardizing tests with water; and they had an outward thrust against them from the compacted fills, and this thrust was not removed by drainages although it was reduced. The bulkhead had a theoretical movement of 0.004 in. (0.1 mm) due to the scale attachments; and, if it did deflect, the actual movement is not known. During the tests, the writer assumed that there was no side-wall movement, and that the possible movement of the bulkhead was too minute to affect the horizontal thrusts of the back-fill materially. However, he must admit that some movement did take place and regrets that it was not actually measured. This change of opinion is due to the experiments made at the Massachusetts Institute of Technology.[31] In his discussion Professor Terzaghi has a made a direct comparison with his Massachusetts experiments, and has shown the similarity between both sets of tests. The ratios found by the Massachusetts experiments are not numerically applicable to soils that differ from the cohesionless, coarse sand used in those experiments, but they do indicate the tendency.

---

[31] "Large Retaining Wall Tests", by Charles Terzaghi, M. Am. Soc. C. E., *Engineering News-Record*, February 1, 1922; March 8, 1929; and April 19, 1934.

[32] *Engineering News-Record*, March 19, 1934, p. 404.

[33] "Vacuum in Soil Mechanics", by H. de B. Parsons, M. Am. Soc. C. E., *Civil Engineering*, December, 1933, p. 690.

Dr. Froehlich believes it probable that the "passive" resistance on drainage accounts for the high values of the horizontal pressures as reported in the paper. When the water was drained, the full hydraulic pressure was released, and the bulkhead might have moved inward, causing the soil to resist being pushed back along its plane of rupture. This reverse action is known as the passive resistance, in contra to the active pressure of the soil wedge forcing the bulkhead outward.

Mr. Thackwell advocates using an "equivalent fluid pressure" method to find the approximate horizontal thrust of a soil wedge on a wall. Usually, this fluid method is based upon the assumption that the material acts like a liquid having a unit weight equal to that of the material, and that the result should be multiplied by a factor determined by experience. For fills without surcharge, this factor is the value of $\tan^2 (45° - \frac{1}{2} \phi)$, and it varies between dry, saturated, and drained conditions. In the Massachusetts experiments, Professor Terzaghi used an hydrostatic pressure ratio, namely, the horizontal component of the soil pressure divided by the horizontal pressure that would be exerted by a liquid the unit weight of which is equal to the unit weight of the back-fill. In other words, the expected soil pressure would equal "$k \times$ liquid pressure". These experiments showed how k varied for slight movements of the wall. It is possible that a series of values may be established in the future for the tangent square, or k, corresponding to varying weights of soils, arranged according to their characteristics, uniformity, dryness, saturation, and wetness (drained), to help obtain results sufficiently accurate for ordinary purposes.

Mr. Thackwell used a depth of 6.979 ft when computing his Table 7. It would have been more consistent to have used the effective head of 6.906 ft, as he used the water pressure at the effective head in the next to the last column of his table.

The writer does not believe that air binding caused the decreasing pressures during the beginning of the irrigations. He thinks it was due to the loss of weight (buoyancy) of the submerged soil and to the increase in the angle of internal resistance of that portion of the soil which was submerged. As the irrigating water rose in the back-fill the hydrostatic pressure increased sufficiently to offset this loss.

Mr. Feld's objection to the converging sides of the bin as causing a decrease of the pressure of the soils on the bulkhead, especially at the sides, is more theoretical than actual. The writer feels that the slope given to the sides materially reduced arching effects, as shown by the consistent results of the repeating tests, and that the benefits from the slope outweighed the disadvantages.

The slope of one side was 0.5 ft in 16 ft, or 0.031 ft per ft of length. The canvas nailed to one side was 0.037 ft in thickness; therefore, the slope of one side did not narrow the back-fill as much as the canvas thickness in the first 1.2 ft as measured on the surface of the fill back from the bulkhead. As a plane of rupture slopes from the surface toward the bulkhead, the narrowing of the back-fill by the contraction of the sides was progressively decreasing from the surface downward. Furthermore, an actual plane of rupture, in a soil having cohesion, is a curved surface concave to the bulkhead; it is not a true plane, and the distance from the bulkhead to the place where a curved rupture plane cuts the surface of a back-fill is less than the distance where the theoretical true plane of rupture would cut the surface. Furthermore, the layer of sand along the side walls, equal to the canvas thickness, helped to give more accurate results than if the bulkhead had been the full width from side wall to side wall. In other words, the layer of sand along the bottom (the ¼ in. below the bottom edge of the bulkhead) and the layers of sand along the sides bounded the wedge causing the pressures by soil and not by the bottom and the sides of the bin.

The writer believes the low values (for sand especially) for the position of resultant pressure were not caused by the shape of the bin, but by a movement of the bulkhead, the effect of which was not anticipated when the apparatus was designed.

Regarding loss of pressure at early stages of irrigation, the writer agrees with Mr. Feld in his statement that "additional moisture content under partial saturation decreased the pressure because of an increase in the coefficient of internal resistance of the material".

Mr. Davey's descriptions of actual failures are most interesting. Masonry is not homogeneous, and a stone wall is not of equal strength throughout its length. A break in a retaining wall would only approximate the theoretical slip of a fill or bank. The illustrations, however, are quite representative of the active forces.

Mr. Davey suggests that possibly the decrease in pressure on first irrigating was due to the compression of the soils, thus permitting them to fall away from the bulkhead. This suggestion is not the writer's idea. The soils were in compacted states, and settlement was not noticed. If any settlement did occur, it was minute. The 5% settlement, to which Mr. Davey refers, was with loose, not compacted, sand.

The writer accepts Professor Krynine's additional conclusion that quantitative results of the "behavior of soil in containers" can be extended to large actual structures only with great care. When accurate test results are correctly interpreted, the conclusions do present knowledge for the design of engineering structures. If the mathematical analysis is correctly applied, using approximately correct data, theoretical and actual results will be close enough to be covered by the usual factor of safety. The rear partition in the bin was too far back from the rupture plane to have had any influence on the pressures on the bulkhead, and the $\frac{1}{4}$-in. layer of soil below the lower edge of the bulkhead helped to remove the "bottom face" and tended to make the tests resemble natural conditions.

The experimental box was made to the dimensions as marked (Fig. 4), which were checked by a tape. The sides were held securely to the dimensions by the tie-rods across the top and by the bottom flooring. Each foot of depth above the bottom was marked on the sides. As the bulkhead was suspended $\frac{1}{4}$ in. above the bottom, the full depth of fill (as measured on the bulkhead) was 6.979 ft. In their discussions, both Dr. Froehlich and Professor Terzaghi state that there was deflection or movement. Measurements were not made to determine deflection of the sides. The bulkhead had a possible movement of 0.004 in. (0.1 mm) due to the scale attachments, and if it did deflect, such additional movement was not measured.

Professor Krynine's idea that surface tension of the capillary water causes the decrease of horizontal pressure at the beginning of irrigation is similar to that of Mr. Wilcoxen, mentioned previously. The tests showed that the angle of internal resistance increased when the soils were irrigated, and Fig. 18 illustrates the writer's idea of the loss of pressure at early stages of irrigation. Assuming for the sake of simplicity that the critical planes are true planes, let *ab* be the plane of rupture of the dry material; *abc*, the wedge causing pressure on the bulkhead; and let *ww* represent the level of the irrigation water at an early stage. The angle of internal resistance of the wetted material, *aw*, increased so that the critical plane of this material would be *ad*, while that of the dryer material above would be *de*. The wedge causing pressure would be *adec* smaller than the original wedge. In addition, the weight of the material in the wedge, *adw*, would be its weight in water. The bulkhead pressure would be increased by the hydro-

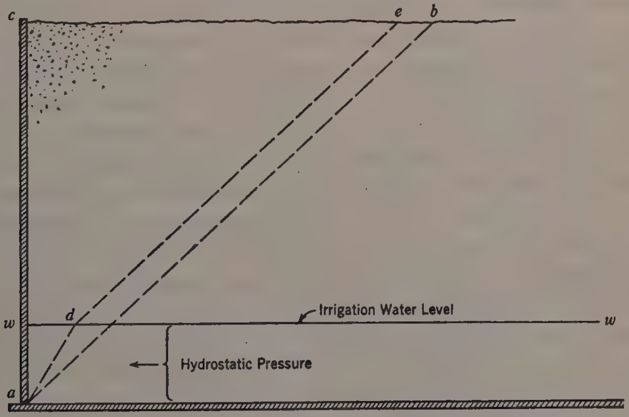

FIG. 18.

static pressure having a head, *aw*. As the irrigation water level rises the total pressure will increase faster than the reduction of pressure due to the soil. This reduction in pressure due to the "soil only" is shown in the horizontal distances, Fig. 3, between the curves marked "Water only" and "Irrigated". The writer has advanced the theory that the angle of internal resistance of an irrigated compacted soil is greater than its angle of internal friction, due to the action of atmospheric pressure.[33]

In the drained material, Professor Krynine believes that the water remaining was not uniformly distributed, but that the water content was more concentrated near the bottom. This is true if the material stands long enough in a drained state. The writer's assumption was made to avoid complication.

Professor Krynine is correct in stating that the calculated angles in Table 4 are angles of internal resistance, rather than of internal friction.

The writer believes that the wedge behind a retaining wall is bounded by a curved rupture plane. He has referred to a true plane only for the sake of more easily studying the pressure phenomena. In Nature, curvature of a rupture plane probably varies according to the amount of cohesion in a soil. A true plane can be conceived for simplicity as one which is an average of the curved surface.

Professor Terzaghi's discussion is of interest because he has explained many phenomena which were puzzling. The experimental apparatus at the Massachusetts Institute of Technology[31] was capable of great refinement, since it had means of measuring, closely, the movement of the wall. Professor Terzaghi's experimental wall weighed 13 tons, while the writer's bulkhead weighed about 800 lb, exclusive of the supporting rods.

The ¼-in. layer of soil below the bulkhead was arranged in order that the lower strata of soil would rest on soil and not on the floor of the bin. The effect of this layer on the results certainly tended toward accuracy.

A comparison with the results of the experiments at the Massachusetts Institute of Technology has been made by Professor Terzaghi and shows a general agreement. The Massachusetts experiments were made with artificially dried, cohesionless sand, having an effective size of 0.54 mm, and an uniformity coefficient of 1.70; while the soils in the writer's tests contained about 5½% of moisture, were not cohesionless, had effective sizes of 0.25 mm for sand, and of 0.57 mm for gravel, and had uniformity coefficients of 5.8 for sand and of 21.0 for gravel.

In the Massachusetts experiments, the value of *k* (horizontal pressure as measured divided by the horizontal pressure of a liquid the unit weight of which is equal to the unit weight of the back-fill) became smaller as the movable wall yielded outward. The bulkhead of the writer's apparatus vibrated while the back-fill was being placed and compacted. To protect the scale bearings and knife-edges, the scale lever arms were wedged. When the bin was filled, these wedges were withdrawn, so that the scales could operate. The movement of the bulkhead was limited to 0.004 in. (or 0.1 mm) which is equivalent to 0.00005 h (in which, h = depth of fill). At the time of testing, it seemed to the writer that this movement was too small to affect the results, but, in the light of Professor Terzaghi's experiments which were made to record the effect of a movable wall, it is probable that a movement of the bulkhead did cause a reduction in the horizontal pressures and in the values of k. The movements were so slight that slips and settlements were not noticed on the soil surface in the bin.

The writer's values for k are 0.22 for loose sand, 0.20 for compacted sand, and 0.16 for compacted gravel. These values are computed from the "corrected" results given for loose sand, and for compacted sand and gravel in Column (4), Table 4, using the rodded weights. The Massachusetts experiments recorded $k = 0.36$ for its loose sand, with an outward tilting movement of the wall of $0.00005\ h$; and for its compacted sand, with the wall yielding outward and parallel to its original position these experiments recorded values of $k = 0.31$ at $0.00005\ h$; $k = 0.21$ at $0.0001\ h$; and $k = 0.18$ at $0.00015\ h$. As the bulkhead described in the paper was hung from links above (Fig. 1), any movement resembled the Massachusetts Test No. 2, in which the wall yielded parallel to its original position.

In the Massachusetts experiments, the centers of pressure were higher than in the writer's tests, but this may have resulted from the fact that the writer's bulkhead moved outward more at the bottom than at the top, whereas the wall of the Massachusetts experiments moved in the reverse direction (see Tests Nos. 1 and 3)[34] or equally at top and bottom (see Test No. 2).

According to Professor Terzaghi,[34] the horizontal pressure ratios, k, depend on the intensity of the frictional stresses that act within the wedges, and the greater these stresses the smaller is the corresponding value for k. This is the same conclusion that the writer's tests illustrated, and the effect of atmospheric pressure may offer a possible explanation, especially for wet soils, as will be explained hereafter.

Six Goldbeck cells were used by the writer, but were not mentioned in the paper, because they did not "make and break" contact sharply so as to give results sufficiently accurate for his research. As Professor Terzaghi used Goldbeck cells in the Massachusetts experiments[35], it would increase general knowledge to record the writer's experience.

The cells were in pairs, three on approximately the center line of the bulkhead and three on one side wall 15 in. back from the bulkhead. The upper pair were 2 ft below the surface of the back-fill, the middle pair, 3 ft 6 in., and the bottom pair, 6 ft. Each cell was set in a mortise and the space around it was tightly packed, making the face of the cell flush with the inner surface of the bin. The air and electric connections entered from the outside, and the pressures were recorded in inches of mercury. The cells were calibrated during the standardizing tests with water, and corrections found for each. It is possible that the corrections for water did not hold true for sand, and, also, as the cells stood vertically, the sand pressures recorded might not have been the true horizontal pressures, but diagonal pressures of an unknown degree.

---

[34] *Engineering News-Record*, February 1, 1934, p. 140.

[35] *Loc. cit.*, February 22, 1934, p. 262.

The pressures, in inches of mercury, as averaged from the records of four dry tests, ten irrigated tests, and eight drained tests of compacted sand, are given in Table 10. The pressures for "sand only" under saturated conditions were found by subtracting from the cell readings for saturated sand the readings for water under the same head as the depth of back-fill. Table 10 gives the readings for the cells on the bulkhead, and on the side wall, as mentioned. The average pressures are plotted in Fig. 19, and the pressure curves drawn. These curves reflect any movement of the writer's bulkhead which may have made the pressures on the bulkhead cells generally lower than those on the side-wall cells (see Table 10).

TABLE 10.—PRESSURE, IN INCHES OF MERCURY, DETERMINED BY GOLDBECK CELLS.

DESCRIPTION OF SAND	DRY			SATURATED SAND ONLY			DRAINED		
Cells	Bulk-head	Side wall	Average	Bulk-head	Side wall	Average	Bulk-head	Side wall	Average
Upper........	1.24	1.33	1.285	0.94	0.88	0.910	1.42	1.31	1.365
Middle.......	1.45	1.86	1.655	0.72	0.88	0.800	1.51	2.66	2.085
Bottom......	2.25	2.45	2.350	0.63	0.70	0.665	2.76	3.17	2.915

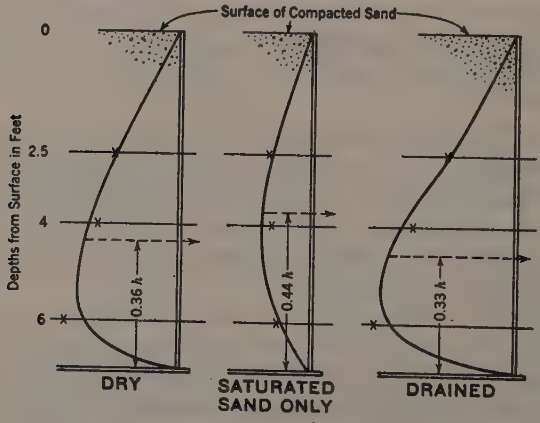

FIG. 19.—PRESSURES AS RECORDED BY GOLDBECK CELLS.

The writer recognized, before reading Mr. Vaughn's discussion, that there seems to be a difference between "internal friction" and "internal resistance", and has used the latter expression in his closure. There seems to be more than "friction" between the grains of a compacted and saturated fill before slipping can occur on a critical plane. Under conditions of a compacted fill that is saturated (not super-saturated) the voids are filled with water, and the writer's theory[33] is that the partial vacua created when the grains tend to, or do, move apart on a plane of rupture cause a portion of the atmospheric pressure to resist movements and to hold the grains from separating. The water in the voids cannot expand (like a gas) when the grains try to move away from their neighbors. However, time and the surface tension of the water may affect the results. Atmospheric pressure is similar to hydrostatic pressure, and acts through the grains of the soil where they are in intimate contact, and also on the water in the interstices between the grains. In consequence, a critical plane, to cause slipping, would have to be steeper for a saturated soil than for the same soil in dry condition; that is, the angle, ϕ, commonly called the "angle of internal resistance", would be greater for saturated than for dry conditions.

To illustrate, let MF (Fig. 20(*a*)), represent a part of a plane of rupture for a dry soil. Draw the vertical, MG, to denote the weight of the soil above M, and GF normal to the plane. Then, MF would be the shear or internal friction, just causing rupture. If the same soil is saturated and partial vacua tend to form at M when the grains above the plane of rupture try to move, a portion of the atmospheric pressure would act along MA, and MX would be its vertical component. Then, lay off XW to denote the weight above M, of the soil in water; and draw the normal, WF_1. The new shear, MF_1, would be less than that required for rupture. If the plane of rupture is made steeper (see MF_1, Fig. 20(*b*)), and MA represents the same unit value for air pressure as before, MX, its new vertical component, and XW, the weight of soil in water as before, the value of MF_1, or shear, will be equal to or greater than MF in Fig. 20(*a*), and rupture will occur. The line, HF_1, represents the horizontal pressure, and will be smaller under saturated conditions than HF under dry conditions in Fig. 20(*a*), which is the result shown by the tests.

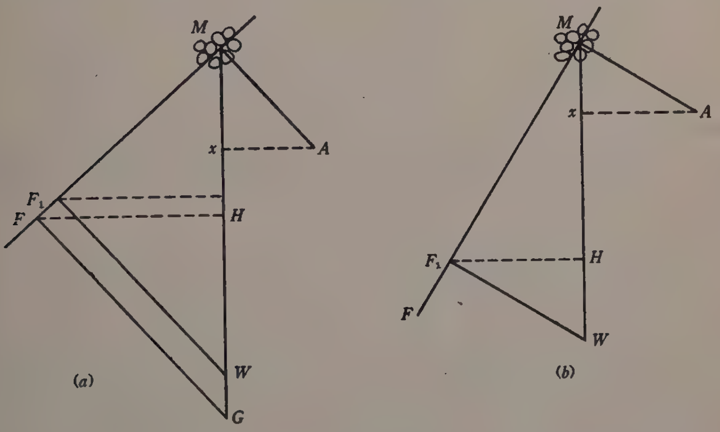

FIG. 20.

The writer has given values to these lines from his test records, and it would appear that the air pressure under saturated conditions which would just cause rupture in Fig. 20 (*b*), would be less than one-quarter of an atmosphere.

From an inspection of Fig. 20, it is seen that, near the surface of a back-fill, the weight, MG, is small and the plane of rupture must be steep to cause slipping. Farther down in the fill the weight, MG, is greater, and the plane can be less steep to create an equal shearing force. This may explain why banks cave on curved surfaces, and why surfaces of rupture for soils are curved and not true planes.

With a saturated back-fill or deposit, against the lower part of the back of a dam which is submerged by the water in the reservoir (the voids being full of water with the grains in a compacted state and not forced apart by super-saturation), it would seem that the same conditions apply as described for a saturated back-fill, the surface of which is exposed to atmospheric pressure. The wedge of the submerged back-fill exerting pressure on the dam would be bounded by its plane of rupture. The total pressure would act with full effect through the mass and would tend to prevent the grains from separating on a normal plane of rupture causing an increase in the size of the voids. Therefore, the plane of rupture would have to be steeper before gravity could separate the grains and slip could occur. If the mass of the fill is super-saturated—that is, if the grains are separated by super-saturation—it is conceivable that the water would act as a lubricant and diminish the angle of internal resistance.

It would seem that atmospheric pressure has a more important part in many soil phenomena than has been recognized heretofore, and its action is more than a relation to capillarity. The tests show that drainages removed the "vacua" support, while subsequent irrigations restored it. The repetitions of loss and recovery of support were very consistent in the tests.

Referring to Mr. Vaughn's discussion of the reduction in horizontal pressure during the first stages of irrigation, the writer mentioned in his paper that the phenomenon was probably caused by the loss of weight of that part of the back-fill which was submerged and to a change in the angle, ϕ, due to the irrigation water. An increase in the angle of resistance would reduce the pressure from the irrigated portion of the fill, both by reduced weight of soil and size of wedge, and would also reduce the size of the wedge of the dry part above. At early stages of irrigation, these reductions might be less than the additional hydrostatic pressure, as explained in connection with Fig. 18.

Mr. Hirschthal mentioned soil-pressure cells, and the writer's results of pressures recorded by the cells used in his research are mentioned herein. In further explanation of Mr. Hirschthal's reference to internal friction, the writer calls attention to the effect of the movement of the bulkhead holding the back-fill, and to the atmsopheric pressure acting internally in the soil mass, thus increasing "internal friction" to "internal resistance". Both these subjects have been discussed herein.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

Discussion

BY MESSRS. J. B. LIPPINCOTT, AND RHODES E. RULE

J. B. LIPPINCOTT,[50] M. AM. SOC. C. E. (by letter)[50a].—The effect of structures on the natural channels in the streams in the southwestern part of the United States, is well described in this paper. The deductions are of interest and value.

Temporary check dams composed of stones wrapped in wire netting and placed on the débris in the canyons or cones, to catch detritus and reduce stream gradients, are a menace rather than a benefit. Following the severe storm on New Year's Day, 1934, in the drainage basin of Verdugo Creek, 12 miles north of Los Angeles, Calif., all such structures inspected, which had not been built on bed-rock, failed, releasing impounded material. They are likely to mislead the public into believing in false safety.

In Section 2 the author states that "the forest cover unquestionably has an important constructive function in Nature which may not be fully appreciated because it is not well understood." The beneficial effect of forests in regulating floods has been generally accepted, but its profound extent has not been appreciated until recent investigations by foresters in California have made available its quantitative effect. A review of some of these determinations appears to be justified. The general conclusion of field tests has been sustained and emphasized by the startling results of the New Year's flood disaster in the drainage basin of Verdugo Creek.

In Southern California stream gradients of the mountain drainage basins are steep and the prevailing crystalline rocks are of a fragmentary and friable nature subject to rapid erosion. The sedimentary formations of the foot-hill areas also are rapidly cut by floods. When exposed to violent storms, floods are projected through the canyons on to the valleys, carrying with

---

NOTE.—The paper by A. L. Sonderegger, M. Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934. by Messrs. H. H. Chapman, and E. B. Debler; April, 1934, by Messrs. Frank E. Bonner, and C. S. Jarvis; May, 1934, by Messrs. W. P. Rowe, and J. C. Stevens; and August, 1934, by Gerard H. Matthes, M. Am. Soc. C. E.

[50] Cons. Hydr. Engr., Los Angeles, Calif.

[50a] Received by the Secretary July 9, 1934.

them surprisingly large quantities of débris which are deposited as deltas cones below the mouths of the canyons. The depth of the valley fills is often in excess of 1 000 ft. Floods travel over these deltas in changing and uncertain channels. Communities located in the plains are menaced both by the floods and also, to a greater extent, by the débris. Occasionally, these violent floods contain surprisingly large percentages of mud, sand, and gravel as a thick emulsion, their high flow marks indicating a discharge too great to appear rational, if they were considered as composed of water only. For instance, in the summer of 1931, a flood occurred in Lone Tree Canyon, in the Tehachepi Mountains, discharging into the Mojave Desert, 14 miles northeast of the Town of Mojave. This was computed from high-water marks, to have been 51 500 cu ft per sec from a drainage basin of 35 sq miles as estimated by engineers of the Department of Water and Power of the City of Los Angeles. This drainage basin rises from 3 000 to 6 708 ft in elevation. The lower half is practically bare. Above 4 500 ft it is sparsely timbered and has little brush cover. Floods exceeding 2 000 sec-ft per sq mile were indicated for certain neighboring basins of from 2 to 4 sq miles. Fig. 3 shows the character of the lower part of the drainage basin and the flood channel of Lone Tree Canyon.

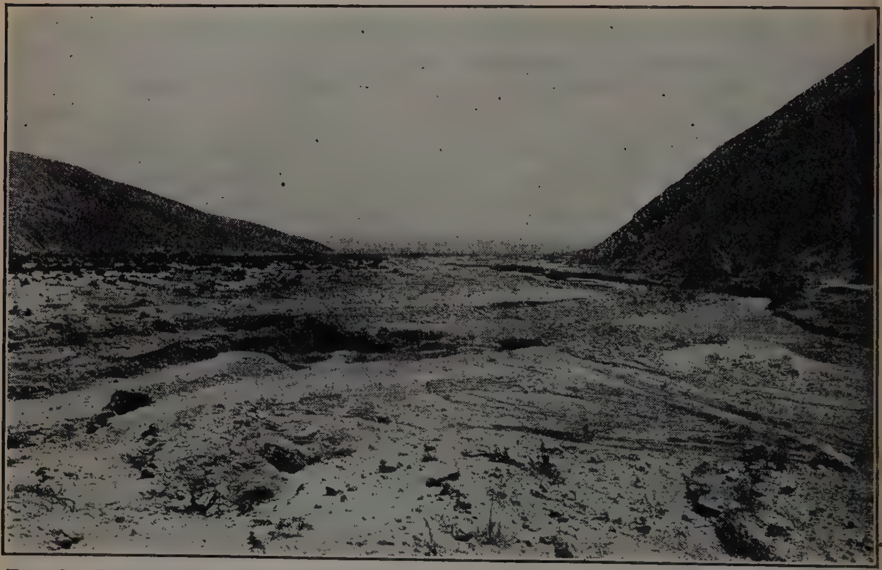

FIG. 3.—LONE TREE CANYON, SHOWING CHARACTER OF CHANNEL FOLLOWING FLOOD OF AUGUST, 1931, WHERE SECTIONS AND GRADES WERE TAKEN. NOTE BARE CHARACTER OF DRAINAGE BASIN.

Another flood on September 30, 1932, in the Tehachepi Pass left marks of indicated flows of 19 200 cu ft per sec from 33.8 sq miles on the headwaters of Tehachepi Creek. This basin is from 3 000 to 5 000 ft in elevation and is about one-half bare of cover. The remaining part has a few scattered oak and pine trees and little brush cover. A flow of 43 500 cu ft per sec was estimated from 20.2 sq miles on Oak Creek.[51] This basin is 10 miles west of Mojave. It rises from 3 600 to 7 950 ft in elevation, and is about three-fourths bare of cover. These three drainage basins are relatively smooth mountain areas. The floods were flashy, resulting from so-called cloudbursts.

During the past few years, the State and National Forest Services have been making intensive studies both in the laboratory and in the field, to find the relationship between water and débris discharge from brush-covered and burned areas.

The storm of December 30 and 31, 1933, and January 1, 1934, which occurred in the Los Angeles area, produced large floods of water and débris from the upper parts of Verdugo Creek. These floods were projected from the steep slopes of the Sierra Madre Range on to the Canada Valley, 12 miles north of Central Los Angeles, causing the death of 34 people, the demolition of 200 homes, and damage to much other property. The results from this storm, accentuated the importance of the study of the foresters of the relation between flows from denuded and forested areas. The data that have been compiled are of such impressive nature to the writer, that it appears of interest to digest some of the records in this discussion.

W. C. Lowdermilk, Senior Silviculturist of the California Forest Experiment Station, at Berkeley, Calif., has conducted a series of laboratory tests for the purpose of throwing light on this relationship. Observations have also been made, under his direction, of the run-off and eroded material from two duplicate small adjacent plots of land at North Fork, on the western slope of the Sierra Nevada, east of Madera, Calif. Two of these plots ($\frac{1}{40}$ acre, each) had normal brush cover, and on two of them this cover was burned. The ground slopes were from 30 to 60 per cent. Recording gauges and sand-traps were used to determine quantities. Dr. Lowdermilk states[52] the following conclusions from this test:

> "Two significant results appear in the data of rainfall and runoff from the plots covered with a natural mantle of vegetation, and from plots burned bare of this vegetation. They are (1) that under intensities of rain up to as much as 2.4 inches per hour no surficial runoff was measured from the covered plots, except where snow drifted into baffle troughs, and (2) that the soil surface burned bare yielded rates of runoff up to 0.30 second feet per acre, representing 200 second feet per square mile. * * * Eroded material from the burned plots reached maximum totals of 3 cubic yards per acre for the rainy season of 1929–30. This material was predominantly mineral soil, whereas material from the covered plots was essentially the organic material of litter."

The eroded material from the burned plots was 241 times as much as from the covered area. One pair of plots is in undisturbed natural forest-brush cover characteristic of the Sierra foot-hills. Two of the plots are on an area on which all cover was cut and burned completely in the fall of 1929.

---

[51] Estimates for Tehachepi and Oak Creeks by the California Div. of Water Resources Rept. (January, 1933), on Flood of September 30, 1932, on Tehachepi Creek.

[52] *Agricultural Engineering*, April, 1931.

Table 6 gives the results of the test for the storm of December 30 and 31, 1933, and January 1, 1934, on these North Fork plots.[58] The total rainfall for the storm was 2.92 in. Its maximum intensities for 10-min and 60-min periods were 0.18 and 0.58 in., respectively. It will be noted that the ratio of run-off for the period of the storm for the area that was burned once, was 14 times that from the unburned area and 463 times that from the area burned annually four times. The eroded material was zero from the unburned area and 100 cu ft per acre for the area that was burned four times.

TABLE 6.—RUN-OFF AND EROSION, IN CUBIC FEET PER ACRE, FROM NORTH FORK EXPERIMENTAL PLOTS; STORM OF DECEMBER 30 AND 31, 1933, AND JANUARY 1, 1934

Area	Unburned	Burned once, 1930	Burned annually 1929–31–32–33
Run-off	4	56	1 854
Ratio	1	14	463
Erosion	0	0.1	100
Ratio	0	1	1 000

Similar observations were made from small plots in San Dimas Canyon, which is on the southern slopes of the Sierra Madre Mountains, in Los Angeles County. These plots are from 1 000 to 1 200 sq ft in area, equipped with inset borders to prevent the incursion of surficial run-off from each side and to prevent the escape of run-off except through the outlets. Automatic measuring devices of the flow are installed. The forest cover consists of brush in the undisturbed plots, the other adjacent plots being burned off. The plots (0.022 acre, each) are established in pairs. The results obtained from observations on the storm of December 30 and 31, 1933, to January 1, 1934, are shown in Table 7. The total rainfall for this storm was 11.68 in.

TABLE 7.—SAN DIMAS RUN-OFF AND EROSION PLOTS; SUMMARY OF STORM, DECEMBER 30, 1933, TO JANUARY 1, 1934

Description	Covered plots		Burned plots	
Plot No.	301	302	303	304
Precipitation per plot, in cubic feet	932.5	932.5	932.5	932.5
Run-off per plot, in cubic feet	1.055	0.702	34.65	37.26
Run-off as percentage of precipitation	0.11	0.08	3.72	3.99
Ratio of run-off	1		40.9	
Maximum rate of run-off expressed as second-feet per square mile	10.2	11.4	223.4	200.7
Eroded material (dry weight, in grams)*	935	446	8 367	16 595
Eroded material (average)	690		12 481	
Eroded material expressed in moisture-free weight, in pounds per acre	68		1 250	
Ratio of erosion	1		18.4	

* Material was dried at 110° C for 40 hr.

---

[58] From paper by Charles J. Kraebel, Senior Silviculturist, California Experiment Station, before a Conservation Conference in Los Angeles, Calif., March 23, 1934.

Its maximum intensities for 5-min, 60-min, and 1 440-min periods were 0.12, 0.97, and 9.55 in., respectively.

The cover in the unburned area consists of a dense growth of brush approximately 10 ft in height. It will be noted that the ratio of run-off from the burned plots was 40.9 times that of the unburned plots, and that the ratio or erosion from the burned plots was 18.4 times the erosion from the unburned plots. These measurements were made by the California Forest Experiment Station.

Table 8 shows the rain that occurred during the storm of December 30, 1933, to January 1, 1934, along the southerly face of the Sierra Madre Range.

TABLE 8.—MAXIMUM RAINFALL INTENSITIES

Station	INCHES OF RAINFALL		Location	Total for storm, in inches
	10-min. period	1-hr. period		
Mt. Lukens	0.30	0.88	Summit of burn	11.04
Haines Canyon	0.27	0.94	West edge of burn	12.08
Flintridge	0.34	1.33	½ mile southeast of burn	14.03
San Dimas Water-Shed	0.25	0.97	25 miles east of burn	10.8
Pomona City	0.28	1.29	30 miles southeast of burn	10.56

While quite uniform it was somewhat unusual in that there was no marked increase in precipitation with rise in elevation. The first station, Mt. Lukens, is at the crest of the Verdugo Water-Shed, Haines Canyon adjoins it on the west and Flintridge to the east. San Dimas is near the base of the range, 26 miles to the east.

TABLE 9.—RESULTS OF NEW YEAR'S STORM; RUN-OFF AND EROSION FROM SOUTHERN SLOPES OF THE SIERRA MADRE RANGE, LOS ANGELES COUNTY, CALIFORNIA

Water-Shed	Rainfall, in inches	Area, in square miles	PERCENTAGE OF WATER-SHED		Peak run-off, in cubic feet per second per square mile	Erosion, in cubic yards per square mile
			Burned	Unburned		
BURNED WATER-SHEDS						
Verdugo *	12.5	19.3	33	67	350	30 700
Pickens †	12.5	0.48	100	0	1 000	50 000
UNBURNED WATER-SHEDS						
Arroyo Seco	12.3	16.24	0§	99.4	58	(No record)
San Dimas	10.8	16.85	0	100	53	56
Fern and Bell ‡	12.4	0.30	0	100	25	52

* Area and run-off computed at Wabasso Way. † Area, etc., computed above junction with east fork. This is a mountainous part of the Verdugo Water-Shed. ‡ Average of four experimental water-sheds, except area, which is total. § The area burned, 58 acres, in backfiring was too small and too lightly burned to affect the run-off from more than 16 sq. miles.

Table 9, taken from Mr. Kraebel's paper,[53] gives run-off and erosion figures from five small neighboring drainage basins. It also shows the extent of burning that has occurred in the Verdugo Basin in the fall of 1934.

The increase in stream run-off and eroded material from the burned range as compared to the unburned areas is startling. The comparison from the small Pickens (burned) Mountain area of 0.48 sq miles and the Fern and Bell (unburned) area is illuminating. The Bell Canyon figures are actual measurements, but the Pickens Canyon quantities are estimated. The estimates by the engineers of the Los Angeles County Flood Control of peak flows from some adjacent small canyons in the burned area were much larger. The rainfall on the different basins was quite similar. The Arroyo Seco Basin adjoins the Verdugo Basin to the east. The San Dimas Basin is 26 miles east of the Verdugo Basin. Their topography and exposure are quite similar except that about 6 sq miles of the Verdugo Basin is the steep absorbent canyon floor. Probably the greater damage to life and property in the Verdugo area was caused by the débris that was projected from the burned areas more than by the water.

The volume of the flows in the Verdugo Basin is not possible of close estimation because of the high percentage of débris carried by the water and the erosion of the channels. It is also difficult to make accurate estimates of the quantity of débris discharged on to the valley floor. It has been computed from surveys as more than 500 000 cu yd, mostly from the burned part of the Verdugo Basin.

On the Arroyo Seco, which adjoins the Verdugo Canyon on the east, the peak run-off of which was caught in a reservoir, there occurred only one-sixth the quantity per square mile of the 19.3 sq miles of the Verdugo, the higher mountainous portion of which had been burned over in the fall of 1933. There is no record of the eroded material from the Arroyo Seco, but it was relatively insignificant. The violence of the flood in the head-waters of the Verdugo Canyon is demonstrated by the size of some of the boulders that were left standing by the storm on concrete street pavements, the largest of these boulders being estimated at 59.5 tons (see Fig. 4). It probably was pushed or rolled along by a mud flow.

It will be noted that only one-third of the Verdugo water-shed was burned over, but this was in a high mountain part of the basin. It will be observed also that La Canada Valley, which has been filled by previous storms with steep sloping débris cones, constitutes another third of the Verdugo Basin. The remainder and unburned part consists of the Verdugo brush-covered mountains and hills. It would be more reasonable to expect less peak run-off per square mile from the Verdugo water-shed had it been brush-covered than from the Arroyo Seco, or other basins to the east, both because of its topography and exposure.

Many "check dams" which consist of rock covered with wire netting, built in the beds of the canyons, to heights of from 6 to 10 ft in the Verdugo water-shed, were destroyed by this flood, demonstrating that such structures, which have been advocated by many, are failures as retarding elements for such floods. If these dams were built on bed-rock and of masonry to heights of 25 to 50 ft, they would have beneficial effect.

Efforts at afforestation in the Sierra Madre Mountains have been failures except on high crests and northern slopes. The brush cover has proved efficient for flood and débris control. It has the advantage of reproducing itself by sprouting from its roots after fires, which is not true of large trees. The so-called "forest reserves" are not used for, and have little value as, a source of lumber.

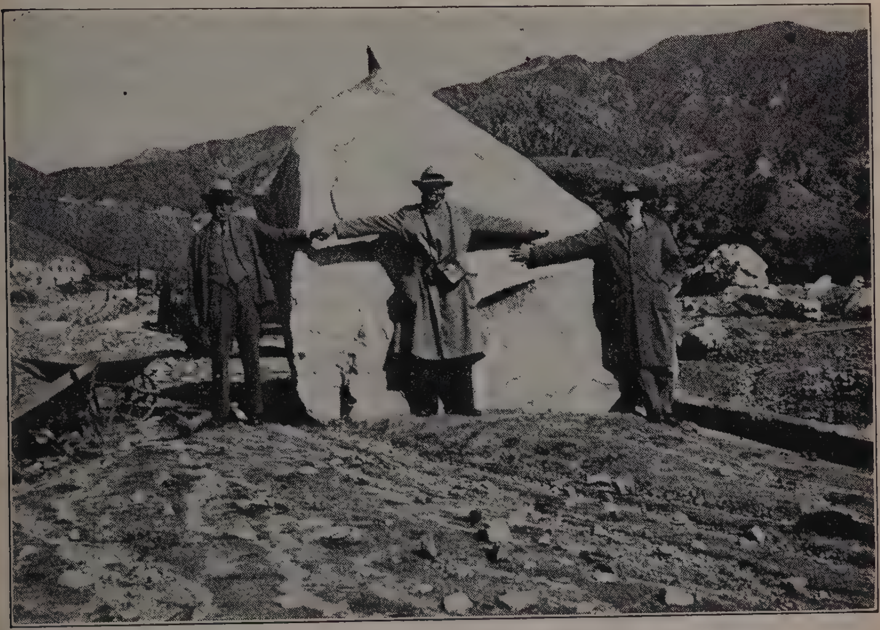

FIG. 4.—FLOOD DAMAGE, MONTROSE DISTRICT, LARGE BOULDER DISPLACED NEAR MOUTH OF DUNSMUIR CANYON. THIS BOULDER IS RESTING ON STREET PAVEMENT.

The lesson to be learned from these floods demonstrates the value of brush cover in controlling floods, preventing erosions, and indirectly aiding in maintaining leveed channels and reservoirs from filling with débris in Southern California. While this a self-evident fact, its full importance has not been quantitatively shown until recently. Greater effort should be made than in the past to preserve the brush cover from fire. Gordon E. Ellis, Assistant Supervisor of the Angeles National Forest, reports that since 1905, 448 fires have occurred in and adjacent to that reserve, which have burned off a total of 227 710 acres, including 102 fires outside the boundaries of the National Forest, the latter totaling 18 704 acres burned. The expenditure by the Federal Government in fighting these fires has been $448 277. In addition to the work of the National Forestry Service, both the State of California and the County of Los Angeles have active organizations which co-operate in forestry work with the Federal Government. Fires occur during the heat of the dry summer and spread through the dense brush with rapidity. It is extremely difficult, if not impossible to extinguish them unless the effort is made at their incipiency. It is essential to reach the fire quickly. Permanent outlooks are maintained in the Forest Reserves and they are also patrolled by airplane to detect fires.

The most effective way of fighting fire is by the construction of a system of roads in the Forest Reserves that will permit of men and equipment being promptly delivered to them when the fire is reported. This can best be accomplished by building trunk roads through the Reserves, which may be used as public highways, from which branch forest roads may be constructed to strategic points, which laterals may be kept closed except in times of emergency. Such road systems are now (1934) being constructed in most of the Southern California Forest Reserves.

The adopted policy of building subestantial masonry dams on the principal stream lines for the purpose of flood control and conservation is desirable but very expensive. They have demonstrated their effectiveness. Usually, the capacities of the reservoirs are small and unless their drainage basins are protected by forest or brush cover so as to prevent large erosion in their tributary basins, the duration of their usefulness is rapidly reduced.

The author has properly called attention to the necessity of careful consideration of stream gradients in relation to constructive conservation works. The protection of the forest cover is a still more basic requirement for the maintenance of permanent flood control.

In Conclusion (3) the author states that "engineers, investigators, and agriculturists have come to realize that the water-shed is their common problem and that co-operation will lead to a rational solution of water conservation in the water-shed." The writer heartily concurs in these sentiments which were endorsed at a large meeting of conservationists, including engineers, foresters, and civic organizations, held in Los Angeles on March 23, 1934.

RHODES E. RULE,[54] ASSOC. M. AM. SOC. C. E. (by letter)[54a].—The result of mature deliberation on an important but usually neglected phase of conservation engineering is presented in this paper. In so doing the author displays that rarest of qualities, the long-range viewpoint. Although many may find exceptions to detailed opinions expressed, none can question the fundamental wisdom of Mr. Sonderegger's proposition.

The author's physiographical balance is not a condition of true equilibrium; it is rather a relative uniformity in the time rate of change of the physiographic condition. As he states, this uniformity is of long-term trend rather than short. However, this uniformity may be over-stressed. Within human experience rivers have made major changes in their courses. Mountains have actually "fallen", forming gigantic natural dams. Other exceptions to the rule that geologic processes are in general non-cataclysmic are abundant. There is a common tendency to exaggerate the importance of normal geologic processes when they occur infrequently.

When going to extremes Nature has a habit of doing a thorough job of it. The late Allen Hazen, M. Am. Soc. C. E., has stated;[55]

---

[54] Civ. Engr., Los Angeles, Calif.

[54a] Received by the Secretary July 10, 1934.

[55] "Flood Flows", by Allen Hazen, p. 53, John Wiley & Sons, New York, N. Y.

"In considering the whole flood problem there is the remarkable condition that a few very large floods are so high in comparison with all the rest that the question is raised as to whether they belong in the same class or whether they are outside and follow some law of their own which comes into operation only at very long intervals and with extraordinary floods."

Recent statistical studies made by the writer tend to bear out this hypothesis that extreme storms—and, hence, floods—follow a law of their own. The tentative conclusion is that great floods are more probable of occurrence than is indicated by ordinary probability studies.

Of recent years it has become the vogue to credit many extreme floods to human interference in the form of fires, timber removal, and over-grazing. The conclusion that removal of this interference will reduce materially the possible maximum flood must be treated cautiously. On the other hand, there is no doubt that these human activities have a marked effect on flood conditions and, in some cases, are the controlling factors.

In the case of the destructive floods of 1923 and 1930, at the westerly base of the Wasatch Range, between Salt Lake City and Ogden, Utah, there is strong geologic evidence to support the theory that the rate of erosion resulting from these floods was much greater than at any time since the passing of Lake Bonneville, a matter of tens of thousands of years. This excessive erosion rate is probably properly attributed to denudation by fire and over-grazing.

Similarly, the La Crescenta-Montrose mud flow in Southern California, on January 1, 1934, was the result of a severe precipitation concentrated on a relatively small but almost completely fire-denuded area of steep watershed. Undoubtedly, some human hand was responsible for setting this fire and was, therefore, responsible for upsetting the physiographic balance which resulted in destructive floods and mud flows, and loss of life and property. It is significant; however, that this took place despite a most elaborate and highly organized system of fire-patrol. This raises the question as to how far human effort can go toward stemming the tide of natural forces.

Before civilized man came on the scene detrital flows of the same nature as those occurring in the La Crescenta-Montrose area were of frequent happenings. In no other way is it possible to account for the alluvial fans at the bases of the Southern California mountain ranges. Detritus hundreds of feet in depth has been deposited in the space of relatively short geologic time. To have accomplished this vast task of transportation required many great floods because it is only in major floods that coarse material, such as that comprising these fans, can be transported in any appreciable quantity. It is doubtful whether works of Man can do much in an economic manner toward controlling such floods to the point that occupation of these alluvial fans can be made safe. Granting that débris dams may be built to accomplish the purpose for a term of years, is the cost balanced by the benefits? Should an entire county be taxed for protecting the property of a small special class of property owners? The major portion of the damage in the La Crescenta-Montrose area was done to improvements lying in the direct paths of countless previous floods.

The author's treatment of the effect of regulation upon the natural balance of a stream system deserves serious consideration. In Southern California the equilibrium between aggradation and degradation is accomplished by many seasons of the former and few of the latter. When storage is provided on a stream for all but the capital floods the burden of establishing a new gradient is imposed upon the stream below the storage. The periods of degradation become much fewer and of less ability to do work of transportation. Add to this the continually growing flood discharges from paved streets in the metropolitan area, and the situation becomes acute. The effect of this discharge from improved areas is to increase the discharge and carrying power of the tributary streams without adding much to that of the main stream. It seems that unless proper precautions are taken the arrival of the rare capital floods will find conditions right for overflow of much highly improved real estate.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MODEL OF CALDERWOOD ARCH DAM

Discussion

BY MESSRS. P. WILHELM WERNER, ELDRED D. SMITH, ELMER O. BERGMAN, AND E. PROBST

P. WILHELM WERNER,[24] ASSOC. M. AM. SOC. C. E. (by letter).[24a]—Many important points pertaining to the design and construction of high dams, and to the application of tests on models of such dams are contained in this paper. Under "Measurements Made on the Model (Temperature Measurements)", the authors state that for the model "it was comparatively simple to keep the temperature variations within such narrow limits that it was not considered necessary to introduce temperature corrections." Referring to Similarity Condition 8,

$$T_m = T_p \frac{c_p}{c_m} \quad \text{..........................(13)}$$

it would be of value to know in what manner the temperature difference, T_m, for the model was, and, for future measurements, will be, controlled in the special case under consideration. Even if it may be fairly safe to assume that the coefficients, c_p and c_m, are sufficiently uniform to be assumed constant, it seems rather difficult to make the temperature difference, T_m, comply with Equation (13) at a sufficient number of points, when considering the irregular distribution of temperatures that occurs in a structure the size of the prototype. Because of the different thermal conductivity, and probably also, the different emissivity, of the materials in the prototype and the model, it would scarcely prove adequate to control only the temperatures of the media surrounding the model.

For a prospective dam there would appear a further complication in that the distribution of the temperatures in the prototype is not known beforehand. This latter obstacle, however, will probably be overcome in the

---

NOTE.—The paper by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1934, by Messrs. A. W. Simonds and Lars R. Jorgensen; and May, 1934, by A. C. Janni, M. Am. Soc. C. E.

[24] Cons. Engr., Stockholm, Sweden.

[24a] Received by the Secretary July 5, 1934.

future through the knowledge obtained by the increasing number of temperature measurements on existing dams and on dams in course of construction.

Similarity Condition 8 will be of importance especially when it comes to determining the effect of the progressive dissipation of the chemical heat in the concrete, and of the changes in the climatic conditions. It is well known (although probably not always fully realized even in the most modern dam designs) that an irregular distribution of the temperature may have a marked influence on, and may even be the dominating factor in, the elastic behavior and stress conditions of a structure of the type in question.

The state of stress in an arch dam due to an irregular distribution of the temperature is very complex in its nature. The behavior of an elementary arch subjected to a differential temperature with a linear distribution between the faces has been referred to elsewhere.[25] Another aspect of the problem is demonstrated in Fig. 16, representing a vertical element of an arch dam with triangular cross-section. Assume that, with the mean temperature kept constant, a certain difference in temperature (ΔT) is introduced between the up-stream and down-stream fibers. (Note that this is not the same as the difference in temperature between the media surrounding the dam, as there will be a drop in temperature also along the contact surfaces between the media and the dam as shown in Fig. 16(*b*)). Whatever the law of transmission of temperature between the up-stream and down-stream fibers may be, it is evident that the difference in temperature will cause a tilting of the cantilever. If $\Delta T < 0$, that is, if the up-stream fiber is cooler than the down-stream fiber, the tilting will be in the up-stream direction, which means that the horizontal arches will be stressed in tension and, probably, the vertical joints open up (if they are not already open). This, in turn, means that the cantilevers will be required to carry a still greater part of the load than otherwise, before the arches take any load. Theoretical considerations, based on some simplifying assumptions as regards the distribution of temperature, will show that this condition may be of great importance in practical cases.

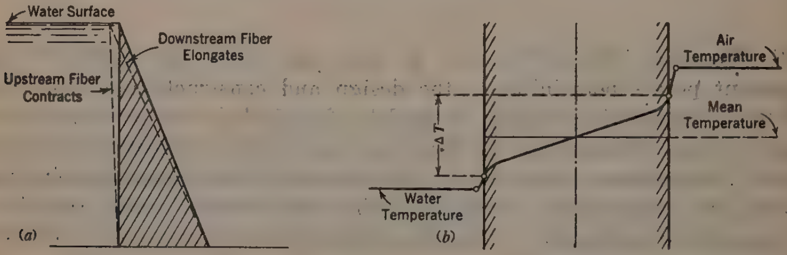

FIG. 16.

When analyzing the results of the measurements on the prototype, it is well to remember the uniform and ordinary elastic properties of the concrete used (see Appendix I of the paper). That the cantilevers in an arch dam built of this material should have tension on both sides simultaneously, due to water load, is very improbable. On the other hand, it is quite possible that an irregular distribution of the temperature, or irregular volumetric changes due to shrinkage, etc., may cause tensile stresses on both sides of a cantilever simultaneously, and these stresses may be of such magnitude that they will outweigh those due to water load and dead load.

---

[25] "Die Staumauern", by N. Kelen, Berlin, 1926.

As far as the writer is aware, the phenomenon of tensile stresses occurring simultaneously on both sides of the cantilever was definitely observed by the authors for the model only. In this case, the question enters as to whether an irregular distribution of the temperature was responsible to some degree for the unexpected results of the measurements. If, on the other hand, the explanation should lie wholly with some peculiar elastic or plastic properties of the model material, then the results obtained when using this material for model study of arch dams may be questioned. This point is of considerable importance, and it would seem desirable that the authors make an attempt to present some theoretical analysis for the explanation of the phenomenon.

That the cantilevers in a concrete arch dam should have excessive tension on the down-stream face due to water load, may also seem unusual and is considered unreasonable by some engineers. In the writer's opinion, however, this condition does not seem impossible although the magnitude of the tensile stresses found experimentally by the authors is rather startling. The so-called cantilever stresses may be looked upon as bending stresses in vertical beams which are supported by an elastic medium constituted by the horizontal arches. It is easily conceivable that under certain conditions contraflexure may occur in the upper parts of the beams. This may be

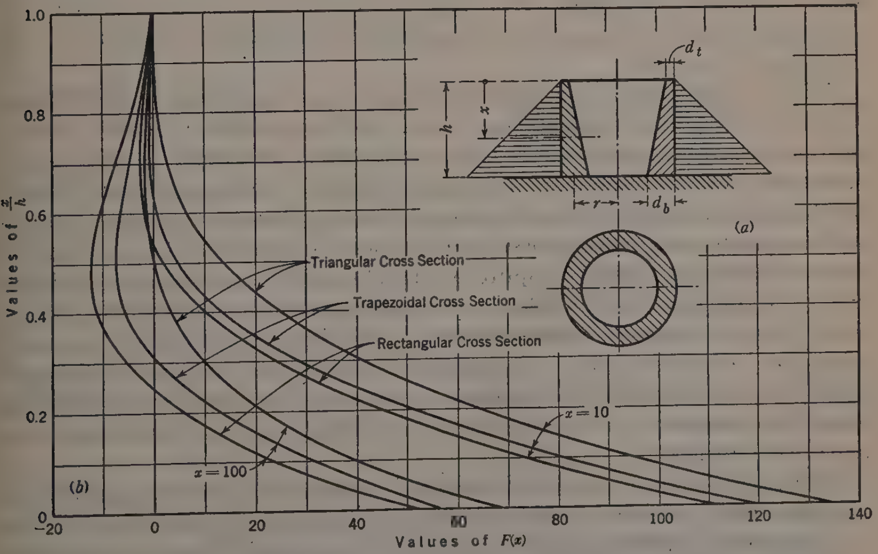

FIG. 17.—CANTILEVER BENDING MOMENTS IN A CYLINDRICAL VESSEL SUBJECTED TO HYDROSTATIC PRESSURE, $M_x = a\,F(x)\,h^3$.

elucidated by another loading case, which is similar in principle, but which is much simpler and, therefore, much more easily and clearly realized than an arch dam. Such a loading case is shown in Fig. 17(*a*), representing a cylindrical vessel subjected to hydrostatic pressure.

The upper edge of the vessel is free like the crest of the arch dam, whereas the lower edge is entirely fixed so as to make the cantilever action more pronounced. Evidently, cantilever tensile stresses on the inner side of this vessel could not be expected any more than on the down-stream face of an arch dam. Nevertheless, it is easily shown that such stresses may occur in the vessel under certain conditions.

The problem of finding the stresses in the cylindrical vessel (bending in the vertical cantilevers and direct stress in the horizontal arches) may be solved, with sufficient accuracy, by a method developed by Dr.-Ing. Th. Pöschl[26], and based on the principle of least work. Pöschl shows that the elastic behavior of the cylindrical vessel is dependent to a considerable degree upon a dimensionless quantity:

$$\kappa = \frac{12\,h^4\,(1 - \mu^2)}{r^2\,d_b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which (see Fig. 17(*a*)), h = the height of the vessel; r, the mean radius of the vessel; d_b, the wall thickness at the bottom of the vessel; and μ, Poisson's ratio.

The bending moment in the cantilevers may be written,

$$M_x = \alpha\, F(x)\, h^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which, α is a constant and $F(x)$ is a function of x. This function has been determined by Pöschl for triangular, trapezoidal ($d_t = 0.5\ d_b$,) and rectangular cross-sections, and is graphically represented in Fig. 17(*b*) for $\kappa = 10$ and $\kappa = 100$.

Fig. 17 is interesting inasmuch as it shows that, under certain conditions, considerable positive moments may occur in the cantilever, causing extensive tensile stresses on the inner face of the vessel. The writer is fully aware of the fact that the arch dam is different from the vessel in that the former has bending moments in the horizontal arches, whereas the latter has no such moments; but, nevertheless, the character of an arch dam (especially the part near the crown) is quite comparable with that of the cylindrical vessel. Consequently it would seem rather natural, under similar conditions, to find cantilever tensile stresses at the down-stream face of an arch dam due to water load. The general dimensions of the Calderwood (arch) Dam also imply that a minute analysis of the dam would probably show this tendency, under the assumption that the dam would act as a monolithic structure.

However, as the grouting of the vertical joints in the dam probably is less than 100% effective, it is scarcely to be expected that the dam will act as a monolithic structure. The vertical elements will then, to a greater xtent than otherwise, act as true cantilevers, with compression or decreased ension on the down-stream side, and tension or decreased compression on he up-stream side. A yielding foundation under the base of the dam would probably have the reverse effect, and thereby would cause an increase in the tensile stresses on the down-stream side. The relative effect of these two factors depends, of course, upon local conditions.

---

[26] "Armierter Beton", by Th. Pöschl, Berlin, 1912, p. 169.

As indicated in the foregoing discussion, the stress distribution may also have been influenced by other factors, namely, temperature changes, swelling, shrinkage, etc. The authors state, that all time effects were excluded from the investigations. In this connection it would be of interest to know more exactly the relative dates of the closing of the dam, the filling of the reservoir, the taking of measurements, and the temperature variations during these periods.

In Appendix I, the authors state that the material in the prototype follows Hooke's law very closely and that it gives the impression of being a very uniform product. Under these conditions the distribution of the stresses could not depart very much from a straight line in the upper part of a triangular element loaded with hydrostatic pressure up to the crest. However, in the cantilevers of an arch dam there are several factors that will cause a redistribution of the stresses: The cross-section may not be exactly triangular; the distribution of the part of the load taken by the cantilevers will be far from the shape of the hydrostatic pressure; etc. The effect of these factors is difficult to determine; but they are of minor importance when the apex angle of the cross-section is comparatively small, so that the "depth" of the cantilevers (base width of dam) can be considered as small in relation to its "span" (height of dam) at any given elevation. In view of this the writer believes that for the upper parts of the Calderwood Dam the assumption of a straight-line distribution of stresses, due to water load, would not involve great uncertainties and, in any case, would cause far less discrepancies between actual and calculated stresses than, for instance, disregarding the stresses due to an irregular distribution of the temperatures.

ELDRED D. SMITH,[27] JUN. AM. SOC. C. E. (by letter)[27a].—A clear and concise statement of the similarity conditions involved in the problem of arch dam model analysis is presented in this paper. It is apparent that the authors are attempting to make a model test by which the action of the prototype can be predicted from the action of the model by applying a single factor of conversion. In attaining this result, it was necessary to violate some of the similarity conditions in order to satisfy others, which may have considerable effect on the final similarity of their results.

In their similarity conditions, the authors state (and go to considerable pains to prove) that the specific gravity of the model and its loading liquid must be in the same proportion as in the prototype. In other words, for a water loading, the specific gravity of the model must be 2.4. However, the manner in which this much emphasized property is used is rather obscure.

---

[27] Junior Engr., U. S. Bureau of Reclamation, Boulder, Colo.

[27a] Received by the Secretary August 4, 1934.

Their results contain no reference to dead load stresses or deflections which are the only stresses or deflections affected by the specific gravity of the structure. Theoretically, this specific gravity requirement is quite desirable, but in order to utilize it, it presupposes the ability to measure simultaneously the dead load strains along with the live load strains as a single quantity. If this cannot be done the advantage of the specific gravity requirement is lost.

In proving the specific gravity requirement the authors state a condition of the principle of superposition:

> "Any increase or decrease in the loading will cause a proportional increase or decrease of the stresses and deflections, provided the deflections are sufficiently small."

The principle in general states that: "The resultant effect of all forces acting on a structure is equal to the sum of the component effects of each force." If the first-named condition holds, the stresses and deflections produced in a structure by a given system of forces, is independent of the stresses and deflections produced by another set of forces. To illustrate, consider the section of a dam as shown in Fig. 18 (not drawn to scale), in which, W = the water load and D, the weight of the dam.

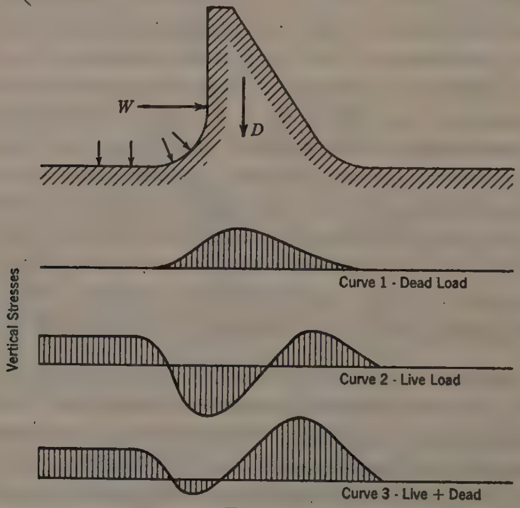

FIG. 18.

Curve 1 shows the stress at the base caused by the weight of the structure; Curve 2 shows the stress at the base produced by hydrostatic pressure; and Curve 3 shows the resultant stress, which is the sum of Curves 1 and 2. Curve 1 and Curve 2 are absolutely independent if the condition of superposition holds. Therefore, the stress distribution due to the water load on the structure is independent of the specific gravity of the material in the structure.

This fact has been proved by three separate experimental investigations of gravity dams on which the writer has been engaged. The models were made of a very light-weight material, but by properly distributed external loads, the equivalent specific gravity was increased to 2.4 × 13.6, or 32.64. The live load liquid was mercury, with a specific gravity of 13.6, so that the specified ratio of specific gravities was maintained. In each of these studies, the effect of the mercury pressure on the structure was the same, whether or not the dead loads were applied. In these tests it was possible to measure the effect due to dead load only, to live load only, and to the combined live and dead loads. Naturally, if the total stress only is to be measured, the theoretical proportion between live load and specific gravity of the structure should be maintained; but, if the dead load stress can be measured as a separate quantity, and the live load as a separate quantity, the experimenter will have considerable latitude in the selection of his model materials, providing the similarity of the elastic properties is maintained.

Consider a model arch dam of a scale, $R = \frac{1}{60}$, with reference to the prototype. The model to be used to predict the stresses in the prototype has a specific gravity of 2.4 and will be loaded with water. All other similarity conditions are assumed to be fulfilled. The ratio of stresses between model and prototype for dead load stresses will be:

$$\frac{1}{R} \times \frac{\rho_p}{\rho_m} \qquad \text{(16)}$$

in which, ρ_p and ρ_m are the specific gravities of the prototype and model, respectively. Equation (16) with the specific gravities of the loading liquids substituted, will give the ratio for the live load stresses. Assume that strains can be measured from a zero or weightless condition (this involves many practical difficulties) so that both dead load and live load strains can be obtained. At a certain point assume that the strain measurements indicate a compressive stress due to dead load only of 10 lb per sq in. and a tensile stress due to live load only of 5 lb per sq in. The stresses in the prototype predicted from these measurements, applying Equation (16) to the model stresses, will be:

$$\text{Dead load} = 10 \times \frac{1}{R} \times \frac{\rho_p}{\rho_m} = 10 \times 60 \times \frac{2.4}{2.4} = 600 \text{ lb per sq in.}$$

and,

$$\text{Live load} = -5 \times 60 \times \frac{1}{1} = -300 \text{ lb per sq in.}$$

This gives a resultant total stress of 300 lb per sq in., compression.

Next, consider the same scale model as before, but loaded with mercury of a specific gravity of 13.6 and, in order to change the proportionality of weights of dam and loading, the material may have a specific gravity of 1.2, or one-half that of concrete. For the same point under consideration, by applying the principle of superposition, the dead load stress in the model will be one-half the previous value, or 5 lb per sq in., compression. By the same token, the live load stress must be 13.6 times that obtained before, or $5 \times 13.6 = 68$ lb per sq in., tension. These stresses will not be referred to the prototype, using the new constants of specific gravity:

$$\text{Dead load} = 5 \times 60 \times \frac{2.4}{1.2} = 600 \text{ lb per sq in., compression}$$

and,

$$\text{Live load} = -68 \times 60 \times \frac{1}{13.6} = -300 \text{ lb per sq in., tension}$$

This gives a resultant total stress of 300 lb per sq in., compression, the same result as obtained before. In this latter case, the specific gravity requirement, as specified by the authors, has been grossly violated, but the stresses predicted for the prototype are the same.

Thus, the question of the fulfillment of a definite specific gravity requirement rests on the ability of the experimenter to measure dead load stresses. It is obvious that to measure the stresses in a structure due to its own weight is a difficult matter. Under "Measurements Made on the Model: Strain Measurements", the authors state:

> "After the model was built, the zero readings, which included the dead load stresses, were taken. The full load readings were taken after the water load was applied. In this way the measured strains represent the strain or stress decrements due to water load only and correspond to the strain measurements on the prototype."

Although the reference to dead load stress given in this statement is not clear, it is plain that the results include only the effect of the water load. Therefore, from the principle of superposition, which states the independence of forces, the writer contends that the authors would have measured the same strains and deflections on their model, regardless of the specific gravity of the material, providing the elastic constants of their material remained the same.

If an attempt to measure dead load stresses in an arch dam model is made, there are two methods which may be followed, without applying external dead loads. The first method is to mount strain-gauges on the layers of material as the model is built so that they will measure the accumulated strain as the height of the model increases. This process would require considerable time, and the accuracy of the results would be affected by: (1) Changes in temperature during the time of construction; (2) the shrinkage or volume changes of the cementing material; and (3), especially in rubber compounds, by the time yield or plastic deformation of the material. Furthermore, if this method were used on the Calderwood model, the strain would be considerably affected by the jacking of the foundation. The second process of determining dead load stresses is a reverse of the first. When all the tests on the model are completed, gauges are mounted on the faces of the model and their change in deflection noted as the model is removed, layer by layer. These readings will be subject to some of the inaccuracies mentioned previously. The time effect will be less and the shrinkage of the cement will have become constant. The latter method was used on a rubber-litharge model on which the writer was engaged.

The apparent goal of the authors was to obtain complete similitude between the action of the model and its prototype; but if the model is to be used as a step in the final design of the structure, this may not be desirable, especially in regard to the construction joints. High tensile stresses in any direction are undesirable in a dam, and the design should eliminate them as far as possible. Therefore, it is more useful to construct the model as a monolith so that no tension will be relieved by the construction joints. In this way, the strain measurements will indicate the areas of tension, and, if necessary, a re-design of the structure may be made to eliminate them. If the model is not constructed as a monolith, the joints will be free to open to relieve tension; therefore, the strain-gauges, unless they should be placed across a joint, would not locate these tension areas. If the gauge were placed across a joint, there would be a question as to how much of the deflection was an actual strain and how much represented a crack opening.

An interesting comparison between two model tests which involved a check with the mathematical theory of arch dams was developed in the laboratory where the writer was engaged. An arch dam model was constructed of a mixture of plaster of Paris and diatomaceous earth and was loaded with mercury to determine the live load stresses and deflections. The trial-load method of analysis was used to determine the distribution of live load between arch and cantilever elements which would produce the measured deflections. Then, as a separate experiment,[28] a model of the crown cantilever alone was constructed to the same scale and of the same material as the model previously tested. This model was loaded by weights with the component of the mercury load which was computed to act on the cantilever element. It was found that this load produced practically the same deflection of the cantilever section as the full mercury load produced in the complete model. The slight difference was accounted for from the larger foundation used in the cantilever model and its freedom from lateral restraint. The most interesting point in this second test was that the cantilever model was loaded artificially by weights acting through pins in the model to produce an equivalent specific gravity of 32.64, so that the relationship specified by the authors was fulfilled; yet the live load deflection of this model checked the deflection of the complete model which was loaded with mercury only.

To summarize this discussion a few points of importance in relation to model studies are offered:

1.—In two-dimensional stress, the stress distribution is independent of the elastic properties of the material, providing Hooke's law holds. Therefore, any elastic material is theoretically suitable for models under two-dimensional stress.

---

[28] "An Investigation of the Crown Cantilever Section of the Boulder Dam by Means of a Plaster-Celite Model", by Charles W. Fletcher, Jun. Am. Soc. C. E.; thesis presented to the Univ. of Colorado in partial fulfillment of the requirements for the degree of Master of Science.

2.—In three-dimensional stress, the stress distribution is not independen of the elastic properties of the materials, but is dependent on the volumetr changes of the material under stress. This requires that Poisson's ratio b the same for both model and prototype if the same distribution of stress to be obtained.

3.—If the effect of external loads only is to be investigated, the specif gravity of the material of the model is not a factor.

4.—If the static effect of the weight of the structure itself is to b measured, only the ratio of the specific gravities of the model and prototyp need be known.

5.—If the combined effect of dead weight and external forces is to b measured as a single quantity, the specific gravity of the model and it external loads must be in the same proportion as in the prototype.

ELMER O. BERGMAN,[29] Assoc. M. Am. Soc. C. E. (by letter)[29a].—Th literature of model testing has been enriched by this comprehensive pape on the Calderwood Arch Dam, and the Engineering Profession is indebted t the authors for their efforts in making available such a complete record c their methods and results. The testing of models of dams is a relativel new field of engineering research and, therefore, it is important that th assumptions, theory, and methods which are proposed be subjected to carefu scrutiny in order that their validity may be established before they ar accepted as the basis for further work in the field.

The first of the questions which the writer wishes to raise has to d with the authors' insistence on Similarity Condition 2 which they state as follows: "The ratio of the specific gravities of the material of th prototype and its loading fluid, $\frac{\rho_p}{\gamma_p}$, must be equal to the same ratio o the model, $\frac{\rho_m}{\gamma_m}$."

If the deflections and stresses to be determined are those due to combine dead and live load, the writer agrees that this is a necessary condition. I would require that the zero readings be taken on a weightless dam and th final readings after the weight of the dam and the water load have bee applied. The authors point out the difficulty of such a procedure. If th strain and deflection measurements are to represent the effect of water loa only, then the dimensional equation[30] reduces to:

$$\delta_p = \psi' (\gamma_p, E_p, g, l) = l_p \frac{(\gamma_p \, g \, l_p)^u}{E_p} \ldots\ldots\ldots\ldots\ldots (17$$

in which, as before, the exponent, u, = 1 and δ_p = the deflection of the prototype; ψ' = an unknown dimensionless function; γ_p = mass per unit volume of the loading liquid of the dam (water); E_p = modulus of elasticit in flexure, of the material of the dam; g = acceleration due to gravity; and l = length scale, so that,

---

[29] Assoc. Prof. of Civ. Eng., Dept. of Civ. Eng., Univ. of Colorado, Boulder, Colo

[29a] Received by the Secretary August 6, 1934.

[30] "Theory of Similarity and Models", by Benjamin F. Groat, M. Am. Soc. C. E. Discussion by A. V. Karpov, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 9 (1932), p. 311.

$$\delta_p = \frac{\gamma_p\, g\, l^2_p}{E_p} \qquad (18)$$

and,

$$\frac{\delta_m}{\delta_p} = \frac{(l_m)^2}{(l_p)} \frac{\gamma_m}{\gamma_p} \frac{E_p}{E_m} \qquad (19)$$

without any requirement that $\frac{\rho_m}{\rho_p} = \frac{\gamma_m}{\gamma_p}$.

The law of superposition may be used to prove that Similarity Condition 2 must be satisfied in order that correct values of combined dead and live load may be obtained. It can not be used to prove that the maintenance of the specific gravity is necessary when only live load effects are to be determined. On the contrary, the law of superposition may be used to prove just the opposite; namely, that any ratio of $\frac{\rho_m}{\rho_p}$ may be used in determining the effect of live load, as long as the condition that the deflections are small is not violated.

A dam is subjected to two different systems of loading. One is the weight of the dam itself, which is proportional to the specific gravity of the material of the dam, and the other is the pressure of the loading liquid, which is proportional to its specific gravity. These systems may be compared to a load, P, at a point, A, in a beam and a load, Q, at a point, B. The deflections and strains in the beam can be measured by taking zero readings before either load is applied and final readings after both loads are applied. Obviously, in accordance with Similarity Condition 2, the deflections and strains due to P and Q could be obtained by applying the loads, $2P$ and $2Q$, and then dividing the measured results by two.

If, however, loads of $2P$ and $3Q$ are applied and, as before, zero readings with both loads off and final readings with both loads on the beam, are taken, there is no way in which the readings can be adjusted to yield the deflections and strains that are produced by the loads, P and Q. On the other hand, if zero readings are taken with both loads off, intermediate readings with the load, $2P$, on, and final readings with both the loads, $2P$ and $3Q$, on the beam, the effect produced by P and Q is equal to one-half the increment of reading, due to $2P$, plus one-third the increment due to $3Q$. The argument is the same as that used in the proof of Maxwell's theory of reciprocal deflections. There is nothing unique about the action of a dam that would serve to exempt it from obeying such a universal principle.

It is difficult to see how Fig. 1 proves anything more than that Similarity Condition 2 holds for combined loads. The diagram itself is likely to be misleading. The area between the solid line, AB, and the dotted line, AB, represents the vertical stress at the foundation line due to an unstated percentage of the dead load. If the vertical distances between the two curves are plotted on a horizontal base line, the curve shown in Fig. 19 is obtained.

This would indicate that the dead load weight of the dam produces tension along the down-stream part of the foundation line, which is unusual to say the least.

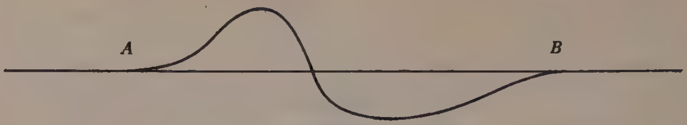

FIG. 19.

The writer also questions the validity of the second reason for considering the relative weights of the loading liquid and the dam material. Provided the deflections remain small, it would seem that doubling the density of the material in the dam would cause a uniform 2:1 increase in the foundation reactions along the entire periphery of the dam.

The third reason seems to be well taken. However, the same change from tension to compression may occur when there is such a marked increase in Poisson's ratio as that from 0.15 to 0.50.

The authors next make the statement (see "Conditions Governing the Properties of the Model Material") that,

> "The only conclusion that can be derived from a model of a gravity dam in which the ratio of the specific gravity of loading liquid to the material is many times higher than in the prototype, is that every gravity dam will fail."

By what process do they arrive at such a conclusion?

To the ten specifications for a material suitable for constructing a model should be added the requirement that the relation of the tensile strength to the compressive strength should be the same as for concrete. This requirement is of major importance.

The authors do not give sufficient weight to the condition that the values of Poisson's ratio should be the same for the material of the model and the prototype. Such easy acquiescence in regard to a factor of major importance is not in keeping with their insistence on the maintenance of the specific-gravity ratio. It is true that a small divergence might be passed over as insignificant in its effect on deflections and stresses, but can such a difference as that between 0.15 and 0.50 be passed over lightly? Moreover, it should be noted that a material with a Poisson ratio of 0.50 presents a special case. For such a value of the Poisson ratio the material deforms with no change in volume, so that if it is compressed at one point, it must expand at another. The deflections, stresses, and strains in a model built of such a material will differ markedly from those in a model built of a material that will undergo considerable change in volume under load, except perhaps for a dam of thin section. It seems doubtful whether reducing the strains in a rubber model to stresses by using a value of Poisson's ratio of 0.15 will give stresses that are even indicative of those in the prototype, as the authors suggest in connection with Figs. 11 and 12.

In connection with the "Uncertainties of the Dam Proper," it is stated that the time interval between the placing of the concrete and the application of the water load causes rather high internal tensile stress to be developed in the blocks of the prototype. Furthermore, the authors state that when the water load is applied, the blocks undergo a contraction under compression which is greater in a block that has a preponderance of internal tensile stress than in one without such stress. Will the authors explain these statements?

In testing the model an unknown factor was introduced by the jacks that were used to change the position and shape of the model and its foundation. The jacks were adjusted until deflection readings at a considerable number of stations were obtained, which corresponded closely with the deflections measured on the prototype, as shown in Fig. 8(*c*). It is then assumed, at least implicitly, that a reasonably close correspondence will exist between the strains and stresses in the model and in the prototype.[31] It is interesting to check the validity of such an assumption when applied to a simply supported beam. Consider a beam, *A*, loaded with a total uniformly distributed load of 5 600 lb and a similar beam, *B*, loaded with a central load of 3 500 lb. The ratio of the deflection of Beam *B* to that of Beam *A* is 1 at the center line and $\frac{55}{57}$ at the quarter-points, while the ratio of the extreme fiber stress is $\frac{5}{4}$ at the center line and $\frac{5}{6}$ at the quarter-points. If loads of 3 200 lb are placed at the quarter-points and a load of 1 300 lb at the center line, the deflections are the same as those of Beam *A* at the quarter-points as well as at the center line; but now the ratios of the stresses in Beam *B* to those in Beam *A* are $\frac{45}{21}$ at the center line and $\frac{11}{6}$ at the quarter-points. Evidently corresponding deflections at five points along the beam do not produce corresponding strains and stresses. There must be correspondence at a sufficient number of points in order that the loads on Beam *B* (model) shall become approximately equivalent to the load on Beam *A* (prototype). Whether the number of points shown in Fig. 8(*c*) is enough to insure reasonable correspondence of strain and stress values between the model and its prototype is at least debatable, particularly in view of the lack of agreement shown in Figs. 11 and 12.

The sum of the horizontal and vertical strains at a point is equal to the sum of the strains in the 45° and 135° directions. This relation may be applied to the data in Table 2 to check their consistency. Deviations may arise from the fact that the measured strains are the average over the gauge line instead of the strain at the point where the gauge lines intersect, and to instrumental and personal errors. Table 2 shows reasonably good agreement. The agreement is better in the model than in the prototype.

With regard to Conclusion (7), the writer believes that a material composed of a mixture of plaster and diatomaceous earth and having a low modulus of elasticity is a better model material than rubber, because its strength properties resemble more closely those of concrete. The chief advantage of rubber as a model material is that it enables strain measurements to be made on the up-stream face.

---

[31] See the authors' discussion of a similar case in "Actual Deflections and Temperatures in a Trial-Load Arch Dam", by A. T. Larned and W. S. Merrill, Members Am. Soc. C. E.; Discussion by A. V. Karpov and R. L. Templin, Members Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1933, p. 1645.

Dr. Ing. E. Probst[32] (by letter)[32a].—The intention of the authors to check the theoretical bases of the design of an arch dam by the use of a model is remarkable in many respects. During recent years a number of measurements have been made on small and larger models of different proposed structures. There are fields, however, in which no direct conclusions can be made based on such tests. Dams belong in that field because they represent structures, the design of which is influenced by the ground conditions to a much greater extent than any other kind of structure. The model can not represent, properly, the condition at the base and abutments of the dam.

In their Conclusion (1), the authors state that a design based on the assumption of non-yielding foundations does not come close to the actual conditions. Differences between the elastic properties of the bed-rock and the material of the dam will change the stress distribution at the base, and that change will increase with the increase of the height of the dam; but the same conclusion can be reached by theoretical considerations.

If, in order to make possible improvements in future designs, it is intended to determine the over-stressed and under-stressed parts of the dam, then the representation in the model of the connections between the dam and the abutments is of particular importance.

The most formidable difficulty in designing the model is the choice of the material, which should have properties different from those of the prototype. The relatively smaller scale of the model makes it impossible to use the same material as in the prototype. The elastic properties and the strength of the concrete are influenced to the greatest degree by the ratio of the different ingredients; these conditions cannot be properly represented even in a large sized model.

Even if it is perfectly mixed and placed, concrete is non-uniform due to the non-uniform distribution of particles of different size. It can be stated that in not a single vertical cross-section of a dam is concrete uniform from the top to the bottom, even if its composition is the same. If the attempt is made to simulate such a complex substance by substituting a uniform material in the model, the conditions of the model will be quite different from those that would be necessary to represent the actual dam. This difference will be in addition to that due to the impossibility of representing, properly, the conditions at the abutments. These considerations and the circular shape of the horizontal cross-section seem to be responsible for the non-uniformity of the horizontal stresses measured on the model. The distribution of horizontal stresses could be improved or could be made uniform by changing this shape.

---

[32] Karlsruhe, Germany.

[32a] Received by the Secretary July 10, 1934.

Quite properly, the authors state in Conclusion (5) that to design an arch dam neglecting gravity action will result in an unwarranted increase in the thickness of the arch at the abutments; furthermore, the evaluated stresses will be quite different from the actual ones.

The authors compare the measurments made on the model with those made on the prototype. In many cases, the deflection measurements may be of some value as an addition to the strain measurements as, for example, in bridges or in structures in which the measuring apparatus is influenced by climatic conditions to a less degree than in dams. The writer knows of cases of deflection measurements on dams which, due to such influences, appeared so unreasonable that they could not be used. Furthermore, all the deflections were so small that, in many cases, even with the greatest possible precision, the small deflection changes could not be determined.

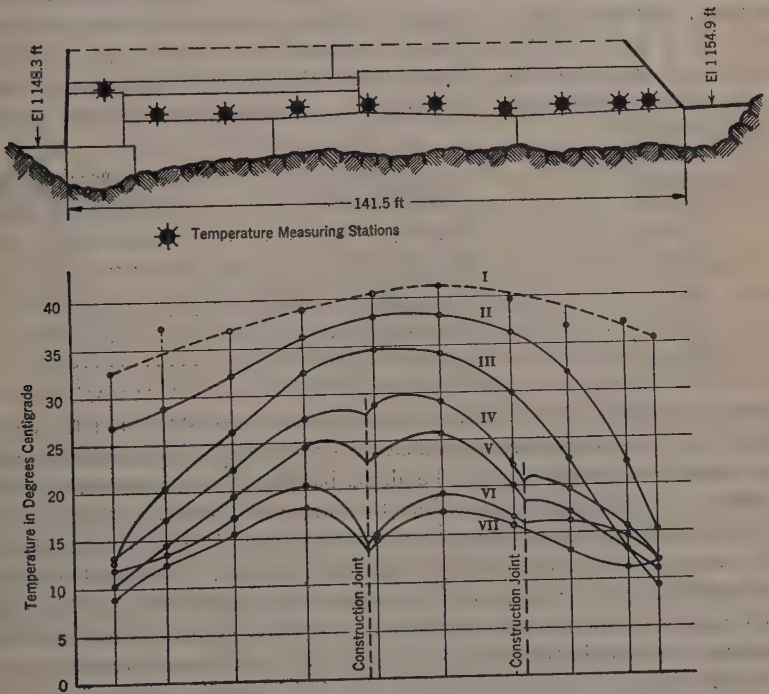

FIG. 20.

The influence of climatic conditions is most pronounced if the strain measurements are made at the faces of a dam as was the case in the measurements made by the authors. Such measurements cannot disclose the strains and stresses within the dam. Even if the measurements made at the faces gave the proper values of the strains, these measurements could not be used to support conclusions regarding the actual stresses in a cross-section of the dam.

All efforts to increase current knowledge concerning the bases of design, necessarily will lead to the necessity of making measurements on the structure itself. Of course, such measurements must be carefully planned based on previous experience. They must be painstakingly accurate, from the beginning of the construction work and must be continued as long as possible after the dam has been put in service. Such measurements can be made only from a distant reference point so arranged as to eliminate the influence of the weather on the measurements.

If it is intended to form a picture of the deformations and stresses within the dam, measurement methods must be used that will disclose the influence of all forces within the thickness of the structure. Besides the influence that is due to the outside and inside temperatures, the stresses at each point will be influenced to a considerable degree by the change in moisture content, due to the continuous setting of the concrete.

A detailed report of such measurements made on the Bleiloch Dam, in Thuringen, Germany (207 ft high), has been presented by the writer elsewhere.[33] At a depth of 197 ft below the crest of this dam, thirty telemeters were inserted at ten stations. Some new data on this subject are shown in Fig. 20, with supplementary information concerning temperature measurements, as follows:

Curve No. (see Fig. 20)	Date of measurements	Time elapsed, in years, since date of first measurement
I	May 15, 1931	0.0
II	October 10, 1931	0.5
III	April 29, 1932	1.0
IV	November 7, 1932	1.5
V	May 5, 1933	2.0
VI	November 6, 1933	2.5
VII	May 24, 1934	3.0

The strains, measured 10 ft above the ground-line joint, show that the changes in strain due to temperature influences are such that no conclusions can be drawn about the strains inside the dam, based on measurements made on the surface. It was shown also that the final conditions in a cross-section were not arrived at three years after the beginning of the construction and nearly two years after the dam was put into service. The final conditions will be reached after some time that depends on the thickness of the dam; but no matter what the conditions, no proper conclusions can be drawn concerning the distribution of stress inside the dam based on measurements made at the face.

The best guaranty of a safe and economical design is to create conditions under which the design assumptions can be controlled clearly and simply. This objective can scarcely be accomplished by model studies.

Comparative measurements on the actual structure should not be limited to surface measurements since the distribution of deformations throughout the cross-section are of the utmost importance as far as the value and location of the unfavorable or objectionable stresses are concerned.

Based on these considerations, the writer would like to suggest that, in the future, measurements be made on each large dam instead of on models.

---

[33] First International Conference on Dams, Stockholm, Sweden.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

AN APPROACH TO DETERMINATE STREAM FLOW

Discussion

BY MESSRS. C. H. EIFFERT, AND CHARLES S. BENNETT

C. H. EIFFERT,[19] M. AM. SOC. C. E. (by letter)[19a].—That the field of study along the line of correlating rainfall and run-off data is far from exhausted, is amply demonstrated in this valuable paper by Mr. Bernard. The author admits that his method has limitations and states in his conclusion what he hopes it may do.

The writer would like to be able to believe that river discharges may be calculated by formula and that regional coefficients may be developed for such a formula, but to the present time his convictions point to quite the contrary.

Stream-flow or run-off formulas should be most valuable for use on drainage areas for which records are not available, or are few. If records are scarce, however, it will be difficult to arrive at the proper coefficient or formula for that region, and, therefore, the solution will be extremely uncertain. Furthermore, the time of year or season will cause a great variation in run-off due to the changes in the capacity of the ground to absorb rainfall. For instance, in the Miami Valley, the run-off from two very similar 1-day rains has varied from 1.25% to 114% of the rainfall.

The writer has not made any extensive use of Sherman's unit-graph method, but believes that the same statement would apply; that is, if records are scarce it will be difficult to arrive at the proper unit graph, especially if 1-day rainfalls from Weather Bureau records are used. Such rains sometimes are spread fairly uniformly over 24 hr, but often they occur in one-half as many hours, or less.

Of course, it must be acknowledged that no run-off figure can be arrived at without some kind of calculation; but in the last analysis the calculations that most nearly approach the correct result will contain a factor which depends largely on the engineering ability and judgment of the calculator.

NOTE.—The paper by Merrill M. Bernard, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. Discussion in this paper has appeared in *Proceedings*, as follows: March, 1934, by C. S. Jarvis, M. Am. Soc. C. E.; April, 1934, by LeRoy K. Sherman, M. Am. Soc. C. E.; and May, 1934, by W. W. Horner, M. Am. Soc. C. E.

[19] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

[19a] Received by the Secretary July 23, 1934.

The best method of attack is through a more thorough and careful study of rainfall intensities and frequencies for the reason that rainfall records are much more voluminous than run-off records.

While engineers are using available records they should concentrate their efforts upon making these records better and more complete in the future. It is true, as stated by Mr. Bernard, that certain unavoidable sources of error must be recognized and accepted in using Weather Bureau data. Certainly, the engineer must recognize them and make the best possible use of the material, but he should not let it go at that. He should exert all possible influence to induce the Government and other departments collecting such information to make it more adaptable for the use of engineers. Considerable can be done along this line without necessarily increasing appropriations. Through better co-operation and co-ordination of data the various Federal Bureaus could make their records more useful to the Engineering Profession without limiting or reducing their usefulness for other purposes. Such matters as making time of readings uniform, obtaining better information on time limits of rainfall, and peak discharges of streams, instead of only the 24-hr maximum, should be easily adjusted. There are other condidtions that could be improved, such as the duplication of gauges by different departments, the breaking off of old records and the starting of others, and the more effective interrelated distribution of the Weather Bureau, Geological Survey, and Army stations.

Another fruitful line of study should be that of time of concentration on various sizes of drainage areas. Few data are available on this subject, except for metropolitan areas. This information will become available, of course, if rainfall periods and peak discharge are properly timed, measured, and recorded.

While studies such as those by Mr. Bernard ought to be encouraged, it must be remembered that nothing can take the place of adequate records. Engineers should be right "on their toes" to see that the errors or deficiencies of the past are not perpetuated.

CHARLES S. BENNETT,[20] M. AM. SOC. C. E. (by letter)[20a].—Great ingenuity and much patience are evidenced in the work of Mr. Bernard in developing the approach to determinate stream flow in this paper. The writer is impressed with the logical development of the plan for constructing the "pluviagraph" and the "distribution graph", and is of the opinion that if basic data were available the method would be of great use to hydraulic engineers.

Unfortunately, there are serious handicaps. The rainfall data of the U. S. Weather Bureau, while complete in a quantitative sense, does not permit of selection of rainfall intensities of less than 24-hr duration. In most instances, there is no way of determining whether a given 24-hr. percipitation occurred in a few minutes, or was distributed over a period of hours. As pointed out by Mr. Bernard, this is a serious drawback in selecting data for the construction of the basic graphs.

---

[20] Engr., The Miami Conservancy Dist., Dayton, Ohio.

[20a] Received by the Secretary July 21, 1934.

The determination of the "water-shed factors" presents practical difficulties. So many coefficients need to be determined, each of which depends largely upon the trained guess of the computer, that a mathematical solution of the equation for determining this factor, U, seems to the writer to be wasted effort.

The application of the method seems limited to streams upon which some previous flow records, including flood flows, are available. This will eliminate the use of the method for many streams.

In the present state of the science of applied hydraulics and meteorological observations, much time would be saved (and likewise laborious calculation) by using simple direct relationships, based on data at hand and judgment, to determine such estimates of stream flow as are necessary for the solution of a given problem.

These criticisms are not intended to deprecate the value of such painstaking research as have been done by Mr. Bernard and numerous other engineers who have been working on the problem of devising means of determining run-off. Great credit is due those who have led the way in attacking the problem. However, many factors involved are indeterminate, and this should be fully recognized by those who attempt to apply the various methods to specific problems.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DISCHARGE FORMULA AND TABLES FOR SHARP-CRESTED SUPPRESSED WEIRS

Discussion

BY W. BRUCE MCMILLAN, M. AM. SOC. C. E.

W. BRUCE MCMILLAN,[8] M. AM. SOC. C. E. (by letter)[8a].—This paper again brings forth the important subject of measurement of water by weirs. The sharp-crested suppressed weir is the type that may be most readily standardized for more precise measurements. If the conditions under which the experiments are made, are duplicated, or made standard, the formulas that represent these experiments accurately may be said to be precise, as limited by the accuracy of observation.

The writer has derived a tentative formula by curve-fitting to substantially the same experiments (Series D to P) as those upon which the paper is based[2] and has arrived at a form similar in general to the Rehbock formula. This same general form may be reduced from the author's formula. Equation (19) may be expressed as follows,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.276 \left[\left(\frac{H'}{H^{\frac{3}{2}}}\right)\right] [1.0195\,(10)^{K}]^{H} \;\ldots\ldots\ldots\ldots\; (32)$$

The first bracket may be represented by:

$$\frac{H'}{H^{\frac{3}{2}}} = 0.9812 + \frac{0.0067}{H} + 0.01209\,H$$

and the second bracket by:

$$[1.0195\,(10)^{K}]^{H} = \left[\,1.017 + \frac{0.095}{P^{1.25}}\,\right]^{H}$$

Substituting in Equation (32) and expanding $(1.017)^{H}$ by the binomial theorem:

$$\frac{Q}{L\,H^{\frac{3}{2}}} \left(3.212 + \frac{0.02405}{H} + 0.0949\,H\right)\left(1 + \frac{0.0934}{P^{1.25}}\right)^{H} \;\ldots\ldots\; (33)$$

NOTE.—The paper by C. G. Cline, Esq., was published in January, 1934, *Proceedings*. Discussion on this paper appeared in *Proceedings* as follows: May, 1934, by Jasper O. Draffin, M. Am. Soc. C. E.

[8] Cons. Engr., Palo Alto, Calif.

[8a] Received by the Secretary July 20, 1934.

[2] "Precise Weir Measurements", by E. W. Schoder, M. Am. Soc. C. E., and the late Kenneth B. Turner, Esq., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 999.

Expanding by the binomial theorem again and casting out the smaller terms:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.212 + \frac{0.02405}{H} + \left(0.0949 + \frac{0.30}{P^{1.25}}\right) H \ldots\ldots\ldots (34)$$

Equation (34) is shown to complete the conversion, but it is not accurate. The studies leading to the writer's tentative formula,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.22 + \frac{0.021}{H} + \left(0.105 + \frac{0.31}{P^{1.33}}\right) H$$

were refined and were adjusted somewhat in deference to the author's determination by least squares, but with deliberate deviation in the case of the highest and lowest weirs for better accord with the experimental data (see Fig. 4). The writer's revised formula is:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.222 + \frac{0.222}{H} + \left(0.081 + \frac{0.340}{P^{1.25}}\right) H \ldots\ldots\ldots (35)$$

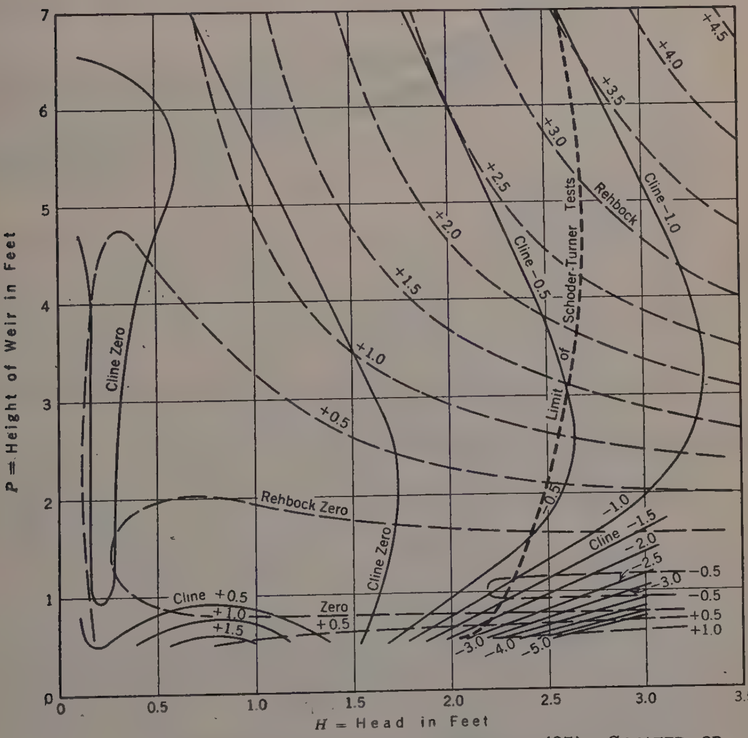

FIG. 4.—PERCENTAGE DISCHARGE BY EQUATION (35), GREATER OR LESS THAN CLINE AND REHBOCK FORMULAS.

In order to compare it with the Bazin and Rehbock equations in the form given by the author (Equations (3) and (4)), Equation (35) is expressed as:

$$\frac{Q}{L} = \frac{2}{3}\left[0.603 + \frac{0.00415}{H} + \left(0.0151 + \frac{0.0636}{P^{1.25}}\right) H\right] \sqrt{2\,g}\; H^{\frac{3}{2}} \ldots (36)$$

Equation (33) gives values of $\frac{Q}{L H^{\frac{3}{2}}}$ almost identical with those by the author's formula. Equation (35) is preferred for greater simplicity in computation as well as being in a more nearly rational form. Fig. 4 shows the percentage deviation of Equation (35) greater or less than by the author's and the Rehbock formulas. Fig. 5 shows a comparison of the same formulas

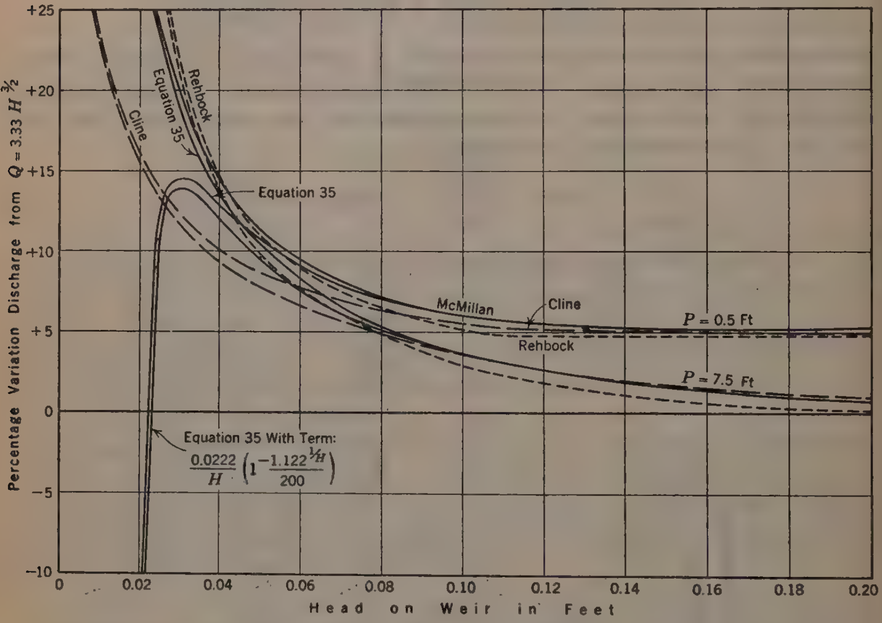

FIG. 5.—COMPARISON OF EQUATION (35) WITH CLINE AND REHBOCK FORMULAS FOR EXTREMELY LOW HEADS.

for weirs 0.50 and 7.50 ft high, for extremely low heads. A curve is also shown for Equation (35) with an additional factor, $\left[1 - \frac{(1.122)^{\frac{1}{H}}}{200}\right]$, as a multiplier to the second term, $\frac{0.0222}{H}$. This factor forms an end point to the curves, where the nappe adheres, and follows the experimental data better for heads less than 0.06 ft; but aside from this it is a matter of interest rather than utility. The conformity of the curves of Fig. 5 is shown by comparison with the curves published by Schoder and Turner, showing measured discharge.[9]

The generalized Francis formula,

$$\frac{Q}{L} = \frac{2}{3}\, C \left[\left(1 + \frac{\alpha\, h}{H}\right)^{\frac{3}{2}} - \frac{(\alpha\, h)^{\frac{3}{2}}}{H}\right] \sqrt{2\, g}\; H^{\frac{3}{2}} \quad\ldots\ldots\ldots\ldots(37)$$

is the fundamental form from which most, if not all, weir formulas are derived and may be shown to be the genesis of Equation (35). Eliminating the second term within the bracket, Equation (37) may be expressed in very general terms,

---

[9] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Fig. 11, p. 1029.

$$\frac{Q}{L} = \frac{2}{3}\, C \left[1 + f(H, P) \right]^{\frac{3}{2}} \sqrt{2\,g}\, H^{\frac{3}{2}} + A + B \quad \ldots\ldots\ldots (38)$$

By a process of curve-fitting and solving for the constants, together with consideration of the studies leading to the original formula and Equation (35), the following is obtained:

$$\frac{Q}{L} = \frac{2}{3}\,(0.602) \left(1 + \frac{0.0683\, H}{P^{1.20}} \right)^{\frac{3}{2}} \sqrt{2\,g}\, H^{\frac{3}{2}} + 0.081\, H^{\frac{5}{2}} + 0.0222\, H^{\frac{1}{2}}$$

or,

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.222 \left(1 + \frac{0.0683\, H}{P^{1.20}} \right)^{\frac{3}{2}} + 0.081\, H + \frac{0.0222}{H} \quad \ldots\ldots (39)$$

Through expansion of Equation (39) by the binomial theorem the form of Equation (35) is obtained. The second or third terms, or both, might be included in the bracket to the $\frac{3}{2}$ power, and the inclusion of these terms was tried with resulting expressions which followed the experimental data quite as well as Equation (39). Professor Rehbock suggests including the term corresponding to $\frac{0.0222}{H}$, ascribing this factor to be attributable to capillarity.[10] As shown subsequently, if this term is maintained outside the bracket, it appears in a form which suggests a crest condition controlled by sub-atmospheric pressure. It is probable that this term is more rational outside the bracket. The term, 0.081 H, is also probably more rational outside the bracket since it is independent of velocity of approach.

Equation (39) gives values of the coefficient within a fraction of a per cent. of those determined by Equation (35), except for low weirs where the values increase in somewhat the same manner as do those of the author's formula, but not to the same degree. Although Equation (39) is more rational, Equation (35) is preferred for simplicity of computation.

Equation (35) may be expressed as:

$$\frac{Q}{L} = 3.222\, H^{\frac{3}{2}} + 0.0222\, H^{\frac{1}{2}} + 0.081\, H^{\frac{5}{2}} + 0.340 \left(\frac{H}{\sqrt{P}} \right)^{\frac{5}{2}} \quad \ldots\ldots (40)$$

This form reveals a combination of simple elements of flow which may be dissected and analyzed. It may be noted, in passing, that Equation (40) lends itself to facility in computation through the use of commonly published tables of squares and the $\frac{3}{2}$ and $\frac{5}{2}$ powers of numbers. The first term, $q_1 = 3.222\, H^{\frac{3}{2}}$, is the theoretical or ideal discharge over a weir of unit width in the familiar form of the Francis formula without velocity of approach, and is the principal term.

---

[10] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Equations (15) to (18), pp. 1148–1149.

The second term, $q_2 = 0.0222\ H^{\frac{1}{2}}$, is the theoretical discharge through an orifice under a head, H, and may be expressed as $q_2 = 0.00277 \sqrt{2\,g}\ H$, which represents the discharge of an orifice, or a 0.00277-ft, or $\frac{1}{32}$-in., slit of unit width, without contraction or friction. This term accounts for an increasing value of the coefficient for heads decreasing from 0.20 ft (see Fig. 5). The term is apparently most affected by the top thickness and by the sharpness of the crest and, no doubt, to some extent by the condition of roughness of the up-stream face of the weir close to the crest. This is indicated by tests of Walker and Weidner[11] and may be explained by the condition of sub-atmospheric pressure between the top of the crest and the nappe, induced by the velocity of the water at this point, and a function of $H^{\frac{1}{2}}$. This reduced pressure and consequent orifice action becomes increasingly effective with an increase in thickness of the top of the crest.

The third term, $q_3 = 0.081\ H^{\frac{5}{2}}$, represents the theoretical discharge without contraction or friction over a triangular weir of constant $\frac{b}{H}$ per unit width, in which, $\frac{b}{H} = 0.0379$ in the familiar formula, $q_3 = \frac{4}{15} \sqrt{2\,g} \left(\frac{b}{H}\right) H^{\frac{5}{2}}$.

The fourth term is the theoretical discharge without contraction or friction over a triangular weir, in which, $\frac{b}{H} = \frac{0.159}{P^{1.25}}$, in the formula, $q_4 = \frac{4}{15} \sqrt{2\,g} \left(\frac{b}{H}\right) H^{\frac{5}{2}}$, and accounts for what is ordinarily referred to as velocity of approach.

These elements of the formula may be represented graphically (Fig. 6), to show a hypothetical distribution of velocity in the nappe. These are not necessarily the actual velocities, but the assumption that they do occur provides an insight to the mechanics of flow over sharp-crested weirs. The third and fourth terms representing discharge over triangular weirs are shown on Fig. 6 to be equivalent increased velocities as measured by the virtual triangular area, to give the same discharge. The parabola, OAX, represents the velocity at any point at which, $v = 0.602 \sqrt{2gy}$, and the area, $OAXY$, represents the discharge, $q = 3.222\ H^{\frac{3}{2}}$. The curve, $OB'Y$, is the hypothetical increased velocity due to the third term, $0.081\ H^{\frac{5}{2}}$, and OBX is the distribution of velocity due to this term and likewise the distribution of velocity when the height of the weir is great, at what is usually referred to as "zero velocity of approach". The curve, $OC'Y$, is the increased velocity due to the fourth term, $0.340 \left(\frac{H}{\sqrt{P}}\right)^{\frac{5}{2}}$, and OCX is the distribution of velocity due to the third and fourth terms, in which, $P = 0.50$ ft, the curves being based on $H = 2.0$ ft. The area, $OCXY$, represents the discharge according to Equation (40) or Equation (35), neglecting the term, $\frac{0.0222}{H}$, which is nominal for a head of 2 ft. The velocity distribution is hypothetical; but the form is suggestive of the forces that act within the jet. Curve *OBX* for a weir of great or infinite height suggests the effect of the factor of curved flow, or the inertia of the stream filaments in curved flow, with special reference to the upward direction of the contraction filaments at the crest. Curve *OBX* is the form of distribution of velocity that would be expected rather than the parabolic distribution, *OAX*. Curve *OCX* for a weir of height, $P = 0.50$ ft, with greater velocity of approach and velocity of the filaments within the jet, indicates the effect of curved flow to be more pronounced. It would appear that this manifestation of velocity of approach does partly, if not entirely, obliterate the effect of the velocity head in the velocity of approach as it is accounted for in the Francis formula, the Fteley and Stearns formula, and in formulas of the Bazin type, including the Frese, Swiss Society, and King formulas.

---

[11] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1046, Fig. 26.

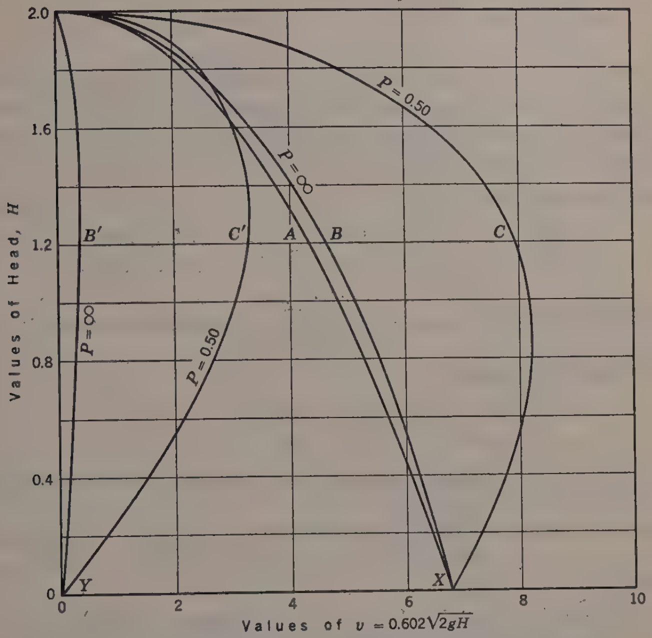

FIG. 6.—HYPOTHETICAL DISTRIBUTION OF VELOCITY IN NAPPE, EQUATION (40).

The velocity of approach factor, $1 + \beta\left(\frac{H}{H + P}\right)^2$, in the formulas of the Bazin type, is derived from the generalized Francis formula (Equation 37), omitting the term, $-\left(\frac{\alpha\, h}{H}\right)^{\frac{3}{2}}$. Expanding by the binomial theorem, substituting $h = \frac{C^2 H^3}{2\,g\,(H + P)^2}$, and assigning a derived and definite value to α and C, or $\alpha\ C^2$, within the bracket, results in the formula,

$$Q = CLH^{\frac{3}{2}}\left[1 + \beta\left(\frac{H}{H + P}\right)^2\right]$$

in which, $C = \left(3.248 + \dfrac{0.079}{H}\right)$ in the Bazin formula and in various forms for others of the same type. As a matter of fact both α and C vary as a function of H and P and the writer's attempt to formulate these values, based on the Schoder and Turner measurements, has led to Equation (39). Casting out a small term involving an error of less than 1%, the Bazin formula may be expressed as:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + \frac{0.079}{H} + H\left[\frac{1.786\,H}{(H + P)^2}\right] \ldots\ldots\ldots\ldots(41)$$

If the envelope of the factor, $y = \dfrac{1.786\,H}{(H + P)^2}$, is substituted in its place, the modified formula will read:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + \frac{0.079}{H} + \frac{0.447\,H}{P} \ldots\ldots\ldots\ldots\ldots\ldots(42)$$

The similarity to the Rehbock formula and Equation (35) is apparent. By introducing the factor, $\dfrac{4\,H\,P}{(H + P)^2}$, the Bazin formula is expressed in a different form:

$$\frac{Q}{L\,H^{\frac{3}{2}}} = 3.248 + 0.079 + 0.447\,\frac{H}{P}\left[\frac{4\,H\,P}{(H + P)^2}\right] \ldots\ldots\ldots(43)$$

When $\dfrac{H}{P}$ is unity in Equation (43), the factor within the bracket becomes unity and Equation (43) is identical with Equation (41). Furthermore, when $H = P$ the factor, $\dfrac{4\,H\,P}{(H + P)^2}$, is a maximum, which accounts for the point of inflection of the Bazin curve and others of the same type (see Fig. 2, for $P = 0.50$ ft). This is also the point of common tangency of the curves for Equations (41) and (42). Due to the fact that formulas of the Bazin type show this inflection while the experimental data show, if anything, a continued curvature or increase of coefficient with head (accounted for in the author's formula and to a lesser degree in Equation (39)), it would appear that formulas of the Bazin type, involving the velocity of approach factor, $\left[1 + \beta\left(\dfrac{H}{H + P}\right)^2\right]$ are incorrect.

It has been advocated many times, and again by the author, that a standard design of suppressed weir be adopted. This is important with reference to the control of the distribution of velocity in a short channel and particular study should be given to the design of a simple means of inducing water in a natural way to a short approach channel if it is not possible to obtain an approach of a length sufficient to insure axial undisturbed flow. A standard distance of point of observation of head becomes increasingly important in the case of high ratio of head over height of weir, due to the water surface draw-down of what approaches the hydraulic drop at the end of a horizontal flume. A standard crest of specific top thickness can readily be duplicated, but there should be no question as to the sufficiency of ventilation beneath the nappe.

The writer has presented several formulas and suggested variations, resulting from three separate studies in derivation. They all reduce most naturally to the general form of Equation (35). This formula, as well as the author's formula and tables, may be applied with confidence for general use. If a high degree of accuracy is required the crest must be truly sharp and $\frac{3}{32}$ in. thick at the top[12] and the approach channel and means of induction of water should conform to that used by Schoder and Turner. However, the importance of the duplication of approach conditions may be overstressed, due to the fact that the water in a normal unobstructed channel tends to adjust itself to a natural regimen on approaching the jet. Inspection of Fig. 4 shows the close conformation of Equation (35) to the Rehbock formula for low weirs. The Rehbock formula is based on careful and comprehensive experiments on weirs from 0.41 ft to 1.64 ft high under comparatively low heads, up to 1.362 ft. Reference to Fig. 2 for $P = 0.5$ ft, shows the Rehbock formula meeting the experimental data as well as, if not better than, the author's formula.

The Schoder and Turner experiments and the formulas based thereon show greater discharge for high weirs (low velocity of approach) than any of the recognized formulas. These older equations have been extrapolated for use not supported by tests. It would appear that it has been the prevailing supposition that high weirs under little or no velocity of approach, under substantial heads, discharge according to parabolic distribution of velocity in the jet—that is, simply at the $\frac{3}{2}$ power of the head. It is indicated herein that the distribution of velocity is reasonably not parabolic and that, due to the action of centrifugal forces in curved flow, the weir discharges an extra quantity above that accounted for by parabolic distribution, determined here to be measured by the term, $q = 0.081\,H^{\frac{5}{2}}$. The possible eccentric approach conditions which may have existed in the run of the Schoder and Turner experiments would be less effective for high weirs with concomitant lower velocity of approach than for low weirs under the same head, and it is reasonable to suppose that the experiments for high weirs are inherently the more accurate. These experiments, complete and comprehensive in range, must be presumed to be correct, at least until they are proved otherwise. The explanation of failure to conform to the recognized formulas is reasonable, and the formulas presented herein are properly substantiated for high as well as for low weirs.

---

[12] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Fig. 3 (Series D to P), p. 1007.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

WILLIOT EQUATIONS FOR STATICALLY INDETERMINATE STRUCTURES

IN COMBINATION WITH MOMENT EQUATIONS IN TERMS OF ANGULAR DISPLACEMENTS

Discussion

BY MESSRS. RALPH A. TUDOR, LEON S. MOISSEIFF,
AND A. A. EREMIN

Ralph A. Tudor,[2] Assoc. M. Am. Soc. C. E. (by letter)[2a].—The complete and accurate analysis of a large suspension bridge tower is an unusual problem and one much more difficult than at first might appear. It is essentially a tall slender column braced in one direction only and subject to enormous vertical loads at its top, which are more often than not acting eccentrically. In addition, horizontal wind loads must be considered and only by comparison with the great vertical loads do they seem small.

The analysis of the tower legs for supporting the vertical loads and for the necessary bending longitudinal with the bridge has been more or less standardized, as described in Mr. Ellis' paper. However, the effect of the participation of the bracing with the legs in supporting the vertical loads, the effect of the transverse wind loads on all parts of the tower, and the magnitude of secondary stresses have not often been considered in detail. The method presented by Mr. Ellis, and his use of the geometry of a Williot diagram covers this phase of the problem and should give results very near the truth. Of great importance is the fact that the method considers, simultaneously, primary (including participation) and secondary stresses. In this manner, allowance is made for the full effect of one on the other without subsequent corrections.

It is unusually interesting that, while Mr. Ellis was finding necessity "the mother of invention," the analysis of another suspension bridge tower was proceeding elsewhere. The same vexing problems were recognized, and a solution was effected which is fundamentally very similar to that already

---

Note.—The paper by Charles A. Ellis, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[2] Senior Designing Engr., Bridges, State Dept. of Public Works, San Francisco, Calif.

[2a] Received by the Secretary March 14, 1934.

presented. The tower differs in architectural detail in that it is braced throughout its entire length, except through the roadway portal, by a system of double diagonals.

Referring to Fig. 15(*a*), it can be seen that a portion of the tower fabricated as shown by the solid lines will be deformed into some such shape as

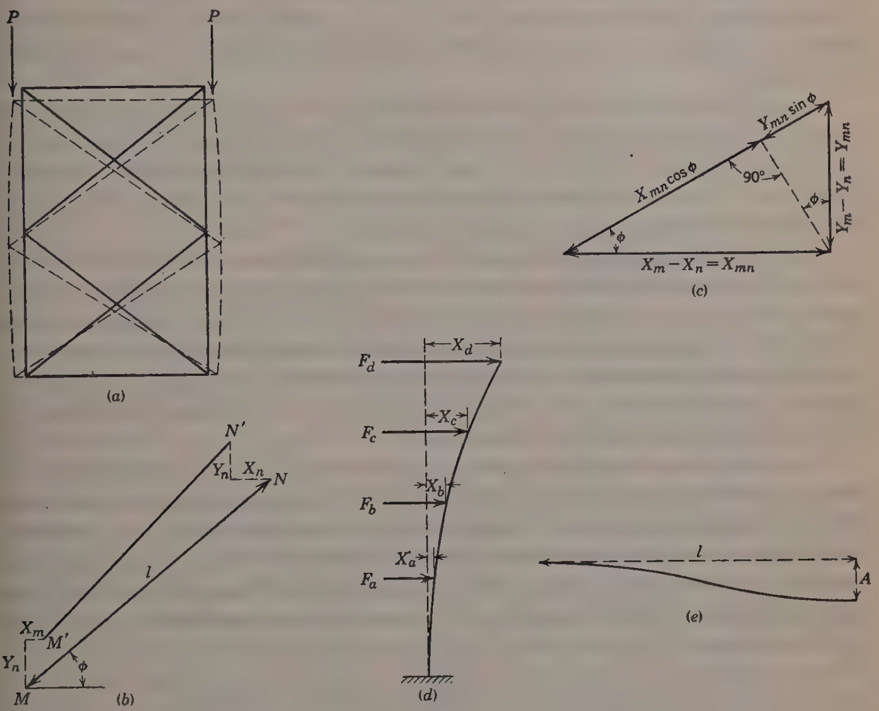

FIG. 15.

that shown by the dotted lines when the loads, *P*, are applied. The elastic shortening of the legs under load will decrease the vertical length of the panels and will require a reduction in the length of the diagonals. This action will place stress in the diagonals and the horizontal component of this stress will be resisted by bending in the tower legs or stress in any horizontal that may exist. Any external horizontal load will create a new stress balance.

When the problem was first encountered it seemed that any solution must be based on the elastic features of the structure, and it was on this premise that a method was developed. The unknowns were the horizontal and vertical (X and Y) components of the movements of the joints. All stresses were expressed in terms of these unknowns and equations of stress balance set up.

Refer to Fig. 15(*b*) and consider any member, such as MN. The unknown components of the movement of Joint M are X_m and Y_m, and those of Joint N are X_n and Y_n. The horizontal component of the length of the member will change by an amount $(X_m - X_n) = X_{mn}$, and the vertical component by an amount $(Y_m - Y_n) = Y_{mn}$. If the slope of the member, ϕ, is assumed to remain unchanged (which is essentially true), the change in length of the member, MN, will be, $\delta_{mn} = X_{mn} \cos \phi + Y_{mn} \sin \phi$ (see Fig. 15(*c*)). The axial stress in the member will then be $\left(\frac{A\,E\,\delta}{l}\right)_{mn}$.

In this manner the axial stress in all members may be expressed in terms of the unknown joint movements. Since joint movements are a feature of the Williot diagram, the method being described is quite similar to that developed by Mr. Ellis.

If all joints were pin-connected and bending of members did not exist, the structure could be solved by equating the sum of the horizontal forces entering each joint to zero ($\Sigma H = 0$), and treating the vertical forces similarly ($\Sigma V = 0$). This would give two equations for each joint, which would equal the number of unknowns and permit a solution.

However, in examining the structure it can be seen that bending in the legs will materially influence the joint displacements and must be fully provided for in the analysis. On the other hand, for the particular tower considered, the web members are comparatively flexible, and their bending will not appreciably effect the distortion of the structure as a whole. Bearing this in mind, equations for $\Sigma V = 0$ may be stated for all joints and for $\Sigma H = 0$ for those joints not along the tower legs.

In stating the remaining equations, the tower legs are considered to be vertical cantilevers fixed at the base. At each joint along the leg a horizontal force of unknown magnitude is applied. This force is simply the algebraic sum of the horizontal components of the axial stresses carried by the members entering the joint, plus any externally applied load. Thus, in Fig. 15(*d*), the leg is acted upon by the forces, F_a, F_b, F_c, and F_d, and the joints are deflected by the amounts, X_a, X_b, X_c, and X_d. Then,

$$\begin{aligned}
X_a &= M_{aa} F_a + M_{ab} F_b + M_{ac} F_c + M_{ad} F_d \\
X_b &= M_{ba} F_a + M_{bb} F_b + M_{bc} F_c + M_{bd} F_d \\
X_c &= M_{ca} F_a + M_{cb} F_b + M_{cc} F_c + M_{cd} F_d \\
X_d &= M_{da} F_a + M_{db} F_b + M_{dc} F_c + M_{dd} F_d
\end{aligned}$$

The M-values are functions of the tower dimensions. By this means the remaining equations are provided, and their solution is only a matter of careful perseverance.

With the magnitude of the joint movements determined for a given condition of external loads, the axial stresses are computed by $\frac{A\,E\,\delta}{l}$. Bending stresses are determined by distributing the fixed end moments in accordance with the Cross method. The fixed end moments (see Fig. 15(*c*)), are equal to, $M = \frac{6\,E\,I\,\Delta}{l^2}$.

This method has been used to solve other frames in which bending of the members is an important item.

LEON S. MOISSEIFF,[5] M. AM. SOC. C. E. (by letter)[5a].—This paper is a contribution of the first order to the methods available to engineers for determining the strains and stresses in elastic frames subjected to considerable deformation and departure from their original geometric form. The author has extended the utilization of the device known as the Willot diagram to the rotation of the members of a stiff frame at any joint. He has opened thereby an approach for the investigator which in many instances will facilitate his labors and in others will afford the only possible path to a closely accurate determination of the behavior of a stiff frame under loads.

Mr. Ellis relates how, encountering the problem of a high tower subjected to large vertical and horizontal forces and designed as a stiff frame, he developed the presented solution. The writer believes that it is pertinent to fix here the co-ordinates of this case in time and space. The Golden Gate Bridge, now under construction (1934), crosses the entrance of San Francisco Bay by a suspended structure with a main span of 4 200 ft and side spans of 1 125 ft. This enormous span, the longest of any bridge, as a matter of design and clearance, requires high towers. They were made about 700 ft in height above the masonry pier and 746 ft above mean high water. The width of the bridge centers of towers, trusses, and cables is 90 ft, and the magnitude of the towers is such that about 21 000 tons of steel are required for one tower.

The location of the bridge over the Golden Gate, the entrance to the largest harbor on the Pacific Coast, in a landscape of rare scenic beauty, demanded a structure of chaste and majestic appearance. It was thought that by avoiding a multiplicity of diagonals for the bracing of the towers and leaving the cross-sectional view of the towers as free from obstructions as possible, a satisfactory solution would be attained. This led to the adoption of a stiff-frame design with diagonals below the roadway and horizontal braces above it. The appearance of the northern tower, which at present is approaching completion, more than realizes the anticipation of the designers.

This is the "flesh-and-blood" story of the skeleton tower which the author encountered and to which he refers. The method and procedure that he developed during his work on the design of the towers are the subject of his paper. It should properly be added that the final proportioning of the towers differs somewhat in cross-sectional areas from those given in the paper. In general, the procedure and method described were followed in the final design of the Golden Gate Bridge towers, on which the author has done excellent work.

The procedure in the final computations was as follows: For transverse wind loads all stresses in the tower were computed simultaneously; the final unknowns were expressed as the rotation angles of the joints and the rotations of the members.

The only simplification introduced in the analysis was in the treatment of the four horizontal struts above the roadway as units. It was made by neglecting the effect of the secondary stresses in the members of the struts on the bending moments in the tower shafts. The shear transmitted through the struts was expressed as a function of the unkown rotation angles of the joints and the end member (tower shaft). It was thus made possible to reduce the number of the final simultaneous equations to thirty, the bottom horizontal member being designed to carry no stress. One tower shaft has eleven joints; the diagonal bracing below the floor has two more; and, there are eleven members in the shaft and six in the diagonal bracing. As each of these represents an unknown, a total of thirty unknowns result. To include the omitted effect of the secondary stresses in the members of the struts in the procedure would have rendered the simultaneous solution nearly impossible. The two upper top struts alone would have added thirty-four additional final equations.

---

[5] Cons. Engr., New York, N. Y.

[5a] Received by the Secretary April 23, 1934.

The secondary stresses in the members of the struts were computed subsequently from the primary stresses, and it was found that the resulting rotation angles at the ends were close to those derived from the solution of the thirty simultaneous equations. It may be added here that the top strut was analyzed subsequently by the same procedure as the tower. This was done in the computations for the tower model mentioned later, as an additional check of the original assumptions.

After the solution of the thirty simultaneous equations, the rotation angles of all joints and members of the shaft were known, as well as the rotation of the end members and end joints of the struts. These values, belonging to the top strut, were then introduced as known quantities, and it was thus possible to compute all stresses in the strut simultaneously by establishing and solving seventeen equations. The results proved that the transferred shear assumed in the original equations was closely correct and that the resulting stresses in the struts were also in agreement with the original computations.

The computations of the stresses and the design of the Golden Gate Bridge which, of course, included that of the towers, have been checked in the writer's office.

Considering the importance and size of the towers and the fact that novel methods of analysis had to be developed for more accurate computations of their stresses, it was decided to verify the design and computations by tests of a model which should simulate, as nearly as possible, the structure and assure elastic behavior within the limits of test loads and observations.

A model of stainless steel was fabricated which was almost an exact replica of the actual tower to the ratio of 1 to 56 in linear scale. The height of the model was 12.5 ft. The vertical load on top of the model was 38 000 lb, corresponding to the vertical load on top of the actual tower of about 120 000 000 lb.

The model was fabricated under the direction of George E. Beggs, M. Am. Soc. C. E., and the tests were conducted under his charge at Princeton University. The model tower was simultaneously loaded both vertically and laterally in proportion to the wind load on the actual tower. A most complete set of observations was made by strain-gauges. The results obtained from the observations on the model were in full agreement with the results of the theory used in the design of the tower.

It will be of interest to know that when the first design of the towers was made, practically for purposes of estimate, it was necessary to determine the lateral deflection of the tower tops. This was computed both by the work method and the slope method. It happened that, by each of these methods, a lateral displacement of about 6 in. was found. Upon closer examination of the problem it became apparent that the deflection found by the work method is due to the axial deformation of the tower shafts only and that the deflection found by the slope method is due to the flexure of the shafts only. Therefore, the two results, the one by the work method and the other by the slope method, should be added in order to obtain the approximate result of displacement. To this again should be added the effect of the eccentricity of the tower loads while so displaced and which will cause additional displacement. The total sum of the lateral displacement thus arrived at was about 17 in. It was then realized that a correct solution could only be obtained by developing a simultaneous integral theory. This has been done by the author. It should be stated here that the lateral deflection of the tower tops is 12.6 in. for the tower as built.

The discussion leads to a comparison of the stiff frame, braced horizontally, with the common diagonally braced frame. It is usually assumed in every-day practice that a diagonally braced frame is statically determinate and that its stresses are therefore definite and computed with little effort. Any one who has given some thought to the problem will admit, of course, that this is not so. In order to have a statically determinate structure, all joints must be hinged and frictionless. Structures are not built in this manner. The frame structures as built are many times statically indeterminate and the correct determination of their stresses is highly complicated.

The assumption of frictionless joints, however, is so prevalent because it is the basis of elementary class-room teaching. This is a necessary result of the evolution of engineering science. Humanity, in its desire to understand the action of natural forces and to forge tools to determine the quantitative character of their action, had to resort to simplifying assumptions. It was an enormous step forward when the conception of the hinged, frictionless frame of rigid members was evolved. To this simplified theory is due the great advance in the design of structures and bridges. For the smaller structures subjected to moderate loads the theory is sufficient. The larger bridges built with higher strength steels and subjected to greater forces and deformations demand a deeper analysis which should correspond more closely to the actual behavior of the structures.

From the point of view of a simultaneous solution a frame with diagonal bracing is more complicated and more laborious to compute than one with horizontal struts. Thus, the Golden Gate Bridge tower, as designed, required thirty equations and, if braced with diagonals, would have required thirty-five equations. The towers of the San Francisco-Oakland Bay Bridge did, in fact, require thirty-five equations.

In connection with the simultaneous analysis of frames and trusses it is well to point out that the separation of strains and stresses caused by external forces, into primary and secondary, is artificial and incorrect. Unless the frame is treated in its entirety erroneous results are frequently obtained. In some cases, even the direction of the stress is in error. There is an evident interaction of the members of a frame and only a simultaneous solution can approach their actual behavior.

A. A. Eremin,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—An original method of computing the stresses in a suspension bridge tower is developed in this paper, including consideration of rigid connections at the joints.

A number of analytical and semi-graphical methods for designing frames have been proposed, the relative merits of which depend upon the error introduced, the simplicity of the operations, and the time saved. Mr. Ellis has developed equations for a direct relation between the angular displacements at the joints and the external forces. However, these equations are complicated and numerous. To compute the angular displacements in the simple symmetrical frame (Fig. 2) below Joints *K* and *L*, fifteen simultaneous equations must be solved. This is a difficult task even with a computing machine.

Furthermore, in the Williot strain diagram, a geometrical relation between small deformations and long normal lines is sometimes difficult to express. Especially is this true in the case of a diagram for an arch frame. Stresses in a frame with rigid joints are generally computed in two operations: The primary stresses are determined under the assumption that the members of the frame are hinged at the joints; and the secondary stresses are computed to express an effect of the rigidities at the joints and eccentricities of the connections on the primary stresses.

Computation of the secondary stresses is based on the convenient formula for a change in the angle due to changes of lengths in all three members in any triangle, thus:

$$E\,\delta\alpha = (S_3 - S_2)\cot\beta + (S_3 - S_1)\cot\gamma \ldots\ldots\ldots\ldots(38)$$

and for checking

$$\delta\alpha + \delta\gamma + \delta\beta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39)$$

The simultaneous equations for computation of the angular displacements developed by Equation (38) may be arranged so that the arithmetical error in solving these equations will not be accumulating. Furthermore, any part of a frame may be analyzed separately.

However, Mr. Ellis has stated correctly that the facility of application of the Williot equations will vary according to the nature and degree of complexity of the problem in hand. The author is to be complimented for this interesting contribution to the theory of design of statically indeterminate structures.

---

[6] Assoc. Bridge Designing Engr. California Div. of Highways, Sacramento, Calif.

[6a] Received by the Secretary, June 26, 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

Discussion

BY MESSRS. L. J. MENSCH, A. A. EREMIN, HANS H. BLEICH, F. H. FRANKLAND, GUSTAV LINDENTHAL, JULIAN W. SHIELDS, A. W. FISCHER, AND J. M. FRANKLAND.

L. J. MENSCH,[20] M. AM. SOC. C. E. (by letter)[20a].—While it is true that Professor Melan used a certain tool of the elastic theory, he knew all the limitations of his assumptions and called his method the "approximate theory" rather than the "elastic theory." He also knew that his formulas for the secondary stresses caused by the deflection of the cables form only an approximate theory and, therefore, called it a more precise theory, and did not use the expression, "exact theory," so liberally found in European technical literature.

The differential equations occurring in this masterly paper by Mr. Steinman are used in many problems which the modern engineer must solve. In the design of thin reservoir and ship plates, Dr. P. Forchheimer[21] shows that one cannot neglect the change of length of the neutral axis of the continuous thin plates, and he treats the resulting suspension action in the same manner as the author, a problem which has been previously discussed by F. Grashof.[22] Dr. Forchheimer simplified the design greatly without the use of any differential equation, and the writer is convinced that Mr. Steinman will be able to simplify his formulas by following Dr. Forchheimer's lead.

The same differential equations occur in the problems of computing the secondary stresses in cylinders held at the ends, such as tanks and boilers, as well as in rectangular tanks to find the stresses in the vertical corners. They occur also in the design of road slabs in finding the stresses due to concentrated loads at the middle or at the edges. All these problems have been simplified so that differential equations are not required for their solution.

NOTE.—The paper by D. B. Steinman, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: August, 1934, by Messrs. Jonathan Jones, A. Müllenhoff, H. Cecil Booth, Jacob Feld, and Glenn B. Woodruff, Howard C. Wood, and Ralph A. Tudor.

[20] Civ. Engr. and Constructor, Chicago, Ill.

[20a] Received by the Secretary July 9, 1934.

[21] "Die Berechnung ebener und gekrümter Behälterböden", von P. Forchheimer, Second Edition, Berlin, 1909.

[22] "Theorie der Elasticität und Festigkeit," von F. Grashof, Berlin, 1878, p. 152.

A. A. Eremin,[23] Assoc. M. Am. Soc. C. E. (by letter)[23a].—Simple equations are developed in this paper, for an accurate calculation of the stresses in a continuous suspension bridge. Furthermore, the author has provided instructive diagrams for the stresses in a suspension bridge continuous over three spans.

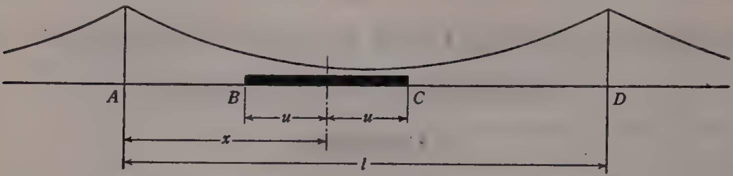

Fig. 10.—Main Span Partly Loaded Near Center.

Fig. 10 may be found convenient in computing the loading terms. For example, the loading terms in Case V, Article 9, may be expressed as follows, to compute values of H, T_1, and T_2:

$$A = pu\left[x\,(l - x) - \frac{u^2}{3}\right] - \frac{1}{c^2}\,B_1 \quad\ldots\ldots\ldots\ldots(65a)$$

$$B_1 = p\left[2\,u - \frac{(1 - d)}{2\,c}\,\frac{(e^{2cu} - 1)\,(e^{2cx} + e^{cl})}{e^{c(x+u)}}\right] \quad\ldots\ldots\ldots(65b)$$

and,

$$B_2 = -\,p\left[\frac{2\,u\,(2\,x - l)}{l} - \frac{(1 - d)}{2\,c\,d}\,\frac{(e^{2cu} - 1)\,(e^{2cx} - e^{cl})}{e^{c(x+u)}}\right] \ldots(65c)$$

For calculating the constants, C_1 and C_2, in Segment BC:

$$G_1 = -\,\frac{p}{4\,d}\left[\frac{(1 - d)\,(e^{2cx} + e^{2cu})}{e^{c(x+u)}} - \frac{(1 + d)\,(e^{2cl} - e^{2c(x+u)})}{e^{c(l+x+u)}}\right] \ldots(66a)$$

and,

$$G_2 = \frac{p}{2}\,\frac{(e^{2cx} + e^{2cu})}{e^{c(x+u)}} \quad\ldots\ldots\ldots\ldots\ldots\ldots(66b)$$

By varying the dimensions, u and x, Equations (65) and (66) may be used for the loading uniformly distributed over any length and at any part in the span. The loading terms for Case VI, Article 9, may be obtained by calculating the terms for the case in which the span is fully loaded and thereafter subtracting the terms calculated with Equations (65) and (66) for loading over the middle part.

Equations (65) and (66) are convenient for computing the loading terms because they contain like factors for all cases of distribution. Furthermore, these equations may easily be simplified for the loading symmetrical about mid-span.

---

[23] Assoc. Bridge Designing Engr., California Div. of Highways, Sacramento, Calif.

[23a] Received by the Secretary July 11, 1934.

The author has developed equations for the stresses and deflections in a suspension bridge with the assumption that the stiffening truss and the suspension cable deflect at any section for the same amount. However, considering the elongation of the hangers, Δh, the bending moment in a stiffening truss will be,

$$M = M_0 - Hy + T - (H + H_w)(\eta - \Delta h) \dots\dots\dots (67)$$

and the differential equation for the deflection of the stiffening truss may be written,

$$\frac{d^2 \eta}{d x^2} = c^2 \eta - c^2 \Delta h - \frac{1}{EI} (M_0 + T - Hy) \dots\dots\dots (68)$$

An elongation in the hanger sustaining the loading from cable and temperature stresses may be expressed as,

$$\Delta h = \left(\frac{H}{r A_h E_h} + \omega_h t \right) (F_c + f - y) \dots\dots\dots\dots (69)$$

in which, A_h = section area of hanger per unit length along the truss; E_h = elastic modulus of hanger; E_c = hanger length at mid-span; and ω_h = coefficient of expansion of hanger.

Substituting Equation (69) in Equation (68) and solving for the deflection of the truss,

$$\eta = \frac{1}{E I c^2} \Bigg\{ C_1 e^{cx} + C_2 e^{-cx} + M_0 - H y + T + E I c^2 \Delta h$$

$$- \frac{1}{c^2} \left[p_x - \frac{H}{r} \left(1 + \left[\frac{H}{r A_h E_h} + \omega_h t \right] \frac{E I c^2}{H} \right) \right] \Bigg\} \dots\dots (70)$$

and the bending moment in the stiffening truss will be:

$$M = - \Bigg\{ (C_1 e^{cx} + C_2 e^{-cx})$$

$$- \frac{1}{c^2} \left[p_x - \frac{H}{r} \left(1 + \left[\frac{H}{r A_h E_h} + \omega_h t \right] \frac{E I c^2}{H} \right) \right] \Bigg\} \dots\dots (71)$$

The constants, C_1 *and* C_2, in Equation (71) are the same as those in Equation (9). Therefore, the moment in Equation (71) differs from the moment in Equation (9) in the term, $\left[\frac{H}{r A_h E_h} + \omega_h t \right] \frac{E I c^2}{H}$. This term has been shown to be small.[24] Therefore, without material error, the effect of the extension of hangers may be neglected as has been stated correctly by the author.

Mr. Steinman has contributed a most outstanding paper to the Engineering Profession. The range of application of suspension bridges is increasing. Therefore, the design method of continuous suspension bridges presented in the paper is timely.

---

[24] "Modern Framed Structures" by Johnson, Bryan, and Turneaure, 1917 Edition. Pt. 2, p. 300.

Dr. Ing. Hans H. Bleich[25] (by letter)[25a].—Following the same idea as that of the author, namely, that suspension bridges with continuous stiffening trusses must have certain advantages over the usual simple-span suspension bridges, the writer has likewise developed a generalized analysis applicable to continuous as well as to non-continuous suspension spans. The writer's procedure is based on the known relationship between the deflection curves of the stiffening truss due to any live loading and the deformation of the stiffening truss due to buckling as a long column under a longitudinal load. This analysis, differing in method of attack, yields final working formulas and numerical results practically equivalent to those obtained by Mr. Steinman, thus affording a substantial check.

One advantage of the writer's method of analysis is that it facilitates consideration of the variable moment of inertia of the stiffening truss. The influence of a varying moment of inertia is particularly important in suspension bridges with continuous stiffening trusses.

The author's claims of superior efficiency for the continuous type of suspension bridge are conservative. He has limited his comparisons to mid-span deflections. By considering also quarter-point deflections, more marked advantages are found even in longer spans.

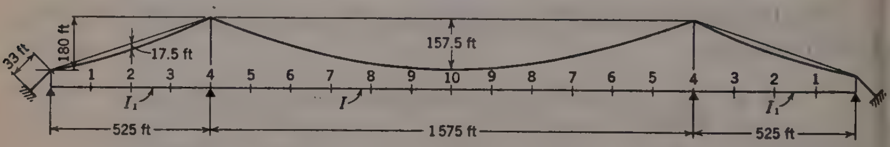

FIG. 11.

The following example of a three-span structure with 1 575-ft main span will show advantages of continuity beyond those pointed out by the author. Referring to Fig. 11, the following general data apply to the problem (using the author's notation): $l = 1\,575$ ft; $l_1 = 525$ ft; $f = 157.5$ ft; $f_1 = 17.5$ ft; $I = I_1 = 33\,400$ in.^{2} ft^2; $A_c = 620$ in.^{2}; $E_c = 23\,500\,000$ lb per sq in.; $w = 13\,440$ lb per ft; $p = 4\,030$ lb per ft; $t = \pm 35°$ C; and $\omega = 0.0000125$.

The maximum bending moment curves for live load and temperature for a continuous truss, and for a two-hinged stiffening truss, both proportioned for the same uniform moment of inertia, are recorded in Fig. 12 (compare with Fig. 5). A comparison of the areas of Designs I and II, Fig. 12, shows that, for the same depth, the continuous truss would require a 6% larger chord section than the two-hinged stiffening truss.

The maximum deflections of the two systems of bridges show that the continuous design (without the 6% addition of chord material) is marked by a notable increase of rigidity over the other system. Table 4 contains the maximum deflections at the center and at the quarter-point of the main span and at the center of the side span. The greatest and most important deflection occurs at the quarter-point of the main span. For this critical deflection, the continuous suspension bridge of 1 575-ft main span, is 17% stiffer than the two-hinged type (although both are proportioned for the same moment of inertia). The corresponding increase in the side-span rigidity is 20 per cent.

---

[25] Vienna, Austria.

[25a] Received by the Secretary June 21, 1934.

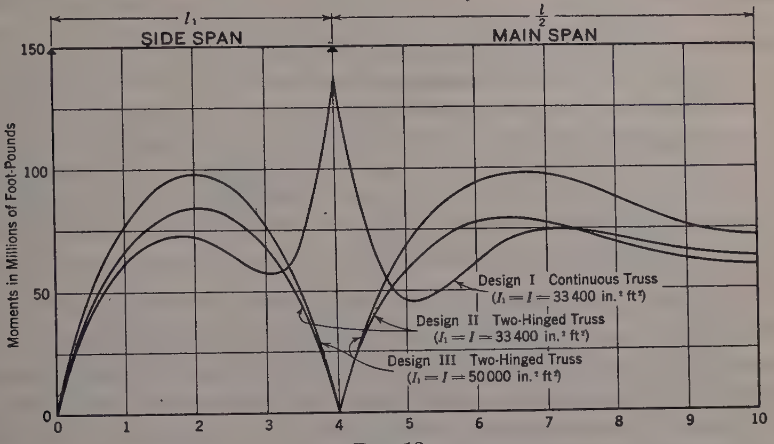

FIG. 12.

The reductions in bending moments from the elastic theory to the deflection theory are found, for this example (see Design I, Fig. 12), to be of similar magnitude to those found by Mr. Steinman for his design with 800-ft main span. These reductions for the design with 1 575-ft main span, amount to about 19% at the tower and about 45% in the main span (compare Fig. 3).

TABLE 4.—COMPARISON OF MAXIMUM DEFLECTIONS, IN FEET (SEE FIG. 12).

Design	SIDE SPAN	MAIN SPAN	
	Center	Quarter-point	Center
I	$1.72 = \frac{l_1}{334}$	$4.11 = \frac{l}{383}$	$3.91 = \frac{l}{403}$
II	$2.15 = \frac{l_1}{244}$	$4.96 = \frac{l}{317}$	$4.03 = \frac{l}{391}$
Percentage difference	20	17	3

A true knowledge of the economic advantage of the continuous type is obtained by comparing the bending moments in two designs of equal rigidity. The moments of inertia of the two designs should be chosen to yield approximately the same average deflections.

Table 5 shows that a two-hinged suspension bridge with $I = I_1 = 50\,000$ in.^{2}ft^2 (Design III), has about the same rigidity as the continuous bridge with $I = I_1 = 33\,400$ in.^{2} ft^2 (Design I). The maximum bending moments from live load and temperature for this bridge (Design III) are also recorded in Fig. 12. The area of this maximum moment graph is found to be 13% larger than for the continuous bridge (Design I). Hence, when proportioned for equal rigidity, the continuous design with 1 575-ft main span is 13% more economical than the two-hinged design. (A comparison of the respective assumed moments of inertia, for equal rigidity, shows an economy of 33 per cent.)

TABLE 5.—COMPARISON OF MAXIMUM DEFLECTIONS, IN FEET, DESIGNS OF EQUAL RIGIDITY (SEE FIG. 12).

Design	Side Span	Main Span	
	Center	Quarter-point	Center
I	1.72	4.11	3.91
III	1.68	4.35	3.78
Percentage difference	—2.5	+6.0	—3.0

The writer has thus confirmed Mr. Steinman's findings of the superior efficiency of the continuous type and has extended these findings to greater span lengths. The same numerical values and comparisons are yielded by either form of the generalized deflection theory. The author would have found the same magnitude and range of superior economy and rigidity if he had considered quarter-point deflections instead of limiting his comparisons to mid-span deflections.

F. H. FRANKLAND,[26] M. AM. SOC. C. E. (by letter)[26a].—The generalized deflection theory of design for suspension bridges presented in this paper, by which the economy realized by the use of the exact or deflection theory, instead of the elastic theory, in the design of bridges with simple-span stiffening girders or trusses, is now available for continuously stiffened, and for multiple-span, suspension bridges. This contribution is undoubtedly one of the outstanding recent advances in bridge design.

The design of suspension bridges has advanced greatly during the past few years, and this development has been much more marked than with other types of bridges. Twenty years ago (1914) it was said with truth that little was known regarding the science of design as relating to this type of structure.

For long crossings there will be many instances where multiple-span suspension bridges will offer the most economic and efficient solution over a considerable range of conditions. These conditions vary from case to case, often in great degree, but in many instances, because of the fact that such structures for highway traffic carry relatively small live loads with fairly large permissible deflections, it is possible to achieve economy, efficiency, practicability, and good appearance in a greater degree with a multiple-span suspension bridge of truly modern design than with any other type. The incorporation of steel floors as a unit with the stiffening girders and wind girder, has permitted hitherto unrealized economies in some recently constructed bridges. Furthermore, in some cases, it is possible to reduce the metal in stringers materially by constructing them continuous over the floor-beams, a feature of special value for self-anchored suspension bridges. When this is done, the stringers become a part of the stiffening system, with corresponding savings in the main stiffening girders.

---

[26] Director, Eng. Service, Am. Inst. of Steel Constr., Inc., New York, N. Y.

[26a] Received by the Secretary July 21, 1934.

GUSTAV LINDENTHAL,[27] HON. M. AM. SOC. C. E. (by letter)[27a].—The generalized deflection theory offered in this paper presents an ingenuously deduced basis for computing stresses in stiffened, suspension bridges whether of one span with end spans, or of multiple spans, assuming continuous parallel-chord trusses through the towers as a recognized feature of economy. It offers a welcome check on other methods of calculation and a refinement toward greater economy, provided the economically most suitable relations of cables and stiffening trusses are first chosen by trial for the given case.

It is a singular fact that, whereas the ascertainment of stresses in a suspension bridge from uniform loads, as for a suspended aqueduct, is a simple matter, that of the stresses from live loads or from moving loads presents certain complexities resulting from the different behaviors of cables and suspended trusses under such loads, further influenced by temperature effects and elastic restraints not wholly statically determinate or subject to elimination by the deflection theory, as, for instance, the elastic elongations of suspenders which for simplicity the author assumes to be negligible.

Therefore, the computed results for any given case, even with the perfected theories now extant, are more of the nature of close approximations than of absolute accuracy, similar to all structures not wholly statically determinate. They are sufficiently accurate, however, for safety and close economy in suspension bridges, which are different from other truss forms in that their carrying capacity is not affected by their deflection under moving loads; only their rigidity is affected, and this can be kept within predetermined limits.

For long spans and concentrated loads it is advisable to make comparisons and checks with models. Suspension structures considered as suspended arches are always in stable equilibrium for any loading. They are unlike the unstable equilibrium of the erect arch in which the center of gravity is far above the points of support, and in which the breaking of a stiffening member may produce collapse; in the suspended form nothing more serious would result than local deformation.

Far in advance for long spans of either the arch or girder types, this anciently known fact early led to the construction of important suspension bridges (600-ft spans over the Menai Strait between the Island of Anglesey and Northwest Wales, in 1832, and over the Danube at Budapest, Hungary, in 1843) stiffened only nominally with hand-rails. Nothing serious can happen to a suspension bridge as long as the anchorage, towers, and cables are sound. In the long-span Brooklyn Bridge, for instance, fractures in the chords of the stiffening trusses, or in the suspenders, have occurred without seriously endangering the structure as a whole.

---

[27] Pres. and Chf. Engr., North River Bridge Co.; Cons. Engr., Jersey City, N. J.

[27a] Received by the Secretary July 13, 1934.

The first theory for the stiffening of a suspended span by the English engineer and mathematician, Rankine (1858), was not used in the United States as far as known, because it gave heavier and more expensive trusses than the empirical Roebling System (low stiffening trusses with stays up to the towers) successfully used for road bridges in this country. The deflection and stresses in the Brooklyn Bridge were studied with the aid of a model which indicated for the long span a sufficiently stiff structure under the exigencies of highway traffic; but a highway and railroad bridge of this system with an 800-ft span, over the Niagara River, did not prove a success. It was a "wobbly" structure, which was later replaced by a rigid steel arch bridge.

The first analysis in the United States of stresses in suspension bridges stiffened with trusses was made by the late Augustus Jay Dubois, M. Am. Soc. C. E.[28] In his treatment of the subject Professor Dubois maintained that the curve of the cable does not remain parabolic as the generally received theory assumed, but that it takes the curve of equilibrium due to loading. He criticized the Roebling System as unscientific and wasteful, and no more bridges of this type were built thereafter.

When Professor Josef Melan, of Prague, Czecho-Slovakia, published, in 1888, his theory of stiffening suspension bridges, he was not aware of the equilibrium theory of Professor Dubois. Professor Melan assumed his stiffening girders hinged at the ends and at the center in order to avoid temperature stresses, but this assumption was corrected in the later editions of his book when the writer had pointed out[29] that the middle hinge does not eliminate temperature stresses but merely re-distributes them.

From the first, Professor Dubois assumed continuous girders fastened down at the ends to resist end bending moments. Both writers assumed only single spans with parallel chord trusses of uniform moments of inertia. Professor Melan's theory, enlarged and supplemented in the later editions of his book, became the accepted theory abroad and in the United States. Other contributors followed with specialized investigations, among whom were the German authors, C. M. Bohny, J. Fritsche, K. Girkman, and F. Glaser. A quite meticulous mathematical treatment of the subject was presented in a new deflection theory (published in English in 1930) by the late Hans Henrik Rode, M. Am. Soc. C. E. Professor Rode's program to extend his method of investigation to suspended and erect braced arches remained incomplete because he died suddenly on July 18, 1930.

Special cases of suspension structures may arise requiring independent investigations. This may happen, for instance, in the possible strengthening of the Brooklyn Bridge for heavier live loads. When this question came up before the writer in 1902, he made a study of different methods of strengthening with the purpose of decreasing the load on top of the stone towers and of transferring part of the load of the superstructure directly to the tower foundations. The stone masonry in the towers is sound, but it is exposed to bending stresses which increase the pressure on the stone to more than 1 000 lb per sq in., and this pressure should be relieved. Additional cables resting on the tops of the towers must be avoided. The study assumed stiffening trusses of greater height and greater moment of inertia at the towers than at the center, or at the anchorages, to take up, through cantilever action, a large proportion of the suspended dead load on both sides of the towers and transfer it directly to the foundations. The arrangement would require a special analysis checked by tests on a model.

---

[28] *Journal*, Franklin Inst., 1882.

[29] Rept. of U. S. Board of Engr. Officers on Maximum Span Practicable for Suspension Bridges, 1894, Appendix D, p. 68, "Temperature Stresses in Three-Hinged Arches", by Gustav Lindenthal, Hon. M. Am. Soc. C. E.

In Section 12 the author discusses the application of his method to multiple suspension bridges with tie-cables connecting the tower tops. Such an arrangement would mar the architectural appearance of a suspension bridge. It would look more like a derrick structure than a bridge, and seems to be avoidable without much loss of economy, at least in highway bridges in which the suspended load is several times heavier than the live load.

The writer bases his belief on his observation of the behavior of a suspension bridge with ten successive spans of 180 ft each under exceptionally heavy street traffic. This was the old Monongahela Bridge, at Pittsburgh, Pa., before it was rebuilt in 1882. Originally, a timber bridge had been built in 1810. When it burned down in 1843, it was rebuilt on the same piers by the late John Augustus Roebling, M. Am. Soc. C. E., as a wire cable suspension bridge of ten spans. Each span was carried by two cradled cables, made on shore, of parallel wrapped wires.

The cables had loops around an iron shoe at each end suspended from short hangers in the top of the cast-iron towers, about 20 ft high and having bases about 8 ft square on the piers. The sag of the cable was one-thirteenth of the span, and on each side was a frail wooden hand-rail of no value as a stiffening element. Heavy horse teams and crowded horse-cars were passing continually in opposite directions. The proportion of live load to dead load was about 1 to 2, and yet the deflections of the roadway under heavy loads were relatively small. The rod iron links from which the cables were suspended in the towers were in constant pendulum-like motion. The pins on which they were suspended were badly worn when the bridge was dismantled.

It was notable that the entire suspended structure of ten spans (total length, 1 800 ft), with all its faults was remarkably steady, and that the time element for the propagation of wave motion in the suspended structure had a deterrent effect on the deflections. If this arrangement was comparatively steady for small spans, it could be made so for long highway spans on which the dead load, including a concrete floor, is several times greater than the live load, and on which the deflection wave must travel longer distances. The better architectural appearance of the bridge may justify the combination of flat catenaries with stiffening trusses continuous through the towers to avoid sagging ties between the tower tops. The widening of tower bases need only be sufficient to include the resultant from the tower top from unbalanced horizontal tension in the cables, which, in a highway bridge, can be kept between narrow limits. Of course, such an arrangement would not do for a railroad bridge, which must be designed to carry concentrated loads at high speeds.

One encounters here, a considerable difference in weight of metal and cost between suspension bridges for more or less distributed highway loads on a flexible structure and that of a structure for the concentrated loads of railroad trains at fast speeds, requiring a rigid track.

In order that a suspension railroad bridge shall be equivalent to a bridge of rigid type (truss, cantilever, or arch type), its local deflections under passing trains should not materially exceed those in these rigid types for equal carrying capacity; but in the suspension bridge stiffened with trusses the weight of steel required for such a purpose may exceed very considerably the weight of the cables which carry the entire dead load and live load, because the size and weight of the stiffening trusses increase approximately in inverse ratio to the predetermined deflection under passing loads, other things being equal.

If this condition would be adhered to strictly, the suspension bridge with stiffening trusses, for the same span, would become nearly as costly for railroad traffic as a bridge of the rigid truss type. The increased cost would not be for increased carrying capacity, but for greater stiffness.

In this respect the study[30] of the late George Shattuck Morison, Past-President, Am. Soc. C. E., for a long-span suspension railroad bridge remains instructive regarding the guiding elements in such a structure stiffened by stiffening trusses. His method of calculation furnished sufficiently close results that would not be affected materially by the author's deflection theory, making due allowance for the higher unit stresses with higher steels that now would be assumed for such a study and that would substantially reduce the weights of metal.

The discussion of Mr. Morison's paper was illuminating in that all the contributors endorsed the decided economy and advantage of stiffening trusses continuous through the tower (the feature adopted also by the author). The discussers also recognized the greater economy of a suspension bridge in the form of suspended braced arches, in which the stiffening web, taken as it were from the trusses, is placed between the parallel cables which act as chords. A criterion of this economy is thus obtained by comparing the weight of steel in each system required merely for stiffening with the weight of the cables.

In his study for a 3 200-ft span, Mr. Morison assumed a total dead and live load of 50 000 lb per lin ft of span. He found the weight of cables was 10 900 lb. The weight of stiffening trusses for a live load of 12 000 lb was 15 780 lb (truss chords, 11 600 lb; web, 3 240 lb; and cross-bracing between top chords, 1 440 lb). In other words, each 100 lb of live load required 90 lb of wire cable and 131 lb of steel purely for stiffening trusses in order to prevent a deflection of the floor in excess of $3\frac{1}{2}$ ft at the quarter-points and to resist the bending and shearing stresses from live load for that deflection. By this comparison the stiffening trusses are, therefore, 31% heavier than the live load and 45% heavier than the wire cables, which carry the entire dead and live load. The weight of the stiffening trusses could be reduced only at the expense of rigidity. Trains would then be compelled to slow up over the bridge, which means it would lose its merit and value as a railroad structure.

---

[30] *Transactions*, Am. Soc. C. E., Vol. XXXVI (1896), p. 359.

On a highway bridge of the same magnitude the live load would be more or less distributed, which would permit a large reduction in the weight of stiffening trusses, eventually less than 25% of the cable weight, without impairment of the capacity and usefulness of the bridge. Traffic regulations can prevent local concentration and congestion, or highway traffic, as is done, for instance, on the Brooklyn Bridge. In this way deflections can be kept within moderate limits; but in a railroad suspension structure in which the deflection wave from passing loads must be kept at a minimum, the weight of the stiffening trusses cannot be reduced arbitrarily without impairing the usefulness of the bridge.

Next, assume a braced suspension arch bridge of the same span in which the web taken from the stiffening truss is inserted between the cables (60 ft apart). Then, suspended braced arches are obtained with a saving assumed roughly at 12 500 lb of steel per lin ft of span over the stiffening truss bridge. Since the suspended dead load in Mr. Morison's study is found to be 39 000 lb, the saving of steel in a suspension braced arch over the stiffening truss type would be 35% on this one item. All values are approximate and are used merely to illustrate the economy of suspended braced arches for the same live loads.

The theoretical treatment of a suspended framed arch bridge is outside the theories for a suspension bridge stiffened with trusses. For the suspended framed arch type, with parallel or nearly parallel chords or cables, the analysis must deal with a structure computable on the theory of elasticity, which for the case of a middle span and two side spans is statically indeterminate in the fifth degree, but with certain permissible assumptions may be reduced to the third degree. When the results are plotted they show that the stresses in the cables from bending moments in the arches can be kept between close limits and require no additional material in the cables. For a completely satisfactory solution checking of the theoretical results on a model is advisable, as is already the practice for large suspension bridges. Such tests will permit verification of computed results and may suggest their further refinement, through application of that part of the author's deflection theory which relates to changes in the curve of the cables from moving loads.

The only suspension bridge type which is statically determinate for moving loads is that with intersecting chords (eye-bar chains), as in the side spans of the Tower Bridge over the Thames, at London, England, or as was proposed in the writer's competing design[31] for the railroad suspension bridge with three spans (659 ft 3 in., 1 758 ft, and 659 ft 3 in.) over the St. Lawrence River, at Quebec, Que., Canada.

The deflections from full live load and temperature (but not from local live loads) are somewhat greater than in equivalent cantilever structures, but are still comparatively small and unobjectionable and even so can occur only rarely. The suspension structure is about 16% cheaper than the cantilever structure, and that its architectural merits are superior to the latter is too obvious for comment.

JULIAN W. SHIELDS,[32] ESQ. (by letter)[32a].—With the publication of this paper, a more general adoption of the suspension type of bridge for relatively short highway spans should follow. The writer has long been interested in the suspension bridge for short spans, recognizing its superior æsthetic properties over the conventional types now in use. More important, an accurately designed suspension structure will prove itself economically justifiable. Actual bidding, in the past, has confirmed the belief that the suspension type can compete successfully with other types in the short-span field.

The major problem of short-span suspension bridge design is to secure sufficient, yet economical, stiffening of the floor system. The hingeless stiffening truss seems best suited to meet the situation. It is admittedly more rigid and, if accurately analyzed, more economical than the two-hinged stiffening truss. Hence, with maximum economy available in the form of the more efficient hingeless stiffening truss, accurately analyzed, the suspension bridge should be able to compete more advantageously than ever before in the short-span field.

A 400-ft span suspension bridge, with 200-ft suspended side spans, was selected in order to study the relative economy of the hingeless and two-hinged stiffening trusses. Following the method outlined in Mr. Steinman's paper, the two truss types were compared, using equal center-span deflections. It is realized that perhaps the truss types should be compared for conditions of equal maximum change in slope. Nevertheless, the results are most gratifying. The hingeless type showed a superior economy of approximately 12% over the the two-hinged truss. This percentage of economy in favor of the hingeless stiffening truss might be increased somewhat if the two truss types were compared under conditions of equal maximum change in grade.

The loadings for maximum moments were found to agree closely with the chart of loadings (Fig. 2) published in the paper. However, there were several exceptions in the example presented by the writer. These exceptions were probably due to the fact that the span lengths adopted were much shorter than those of the example given by Mr. Steinman. This might be expected, as the author indicates. The maximum moments at the sections,

---

[31] *Engineering News*, November 23, 1911.

[32] Instr. in Civ. Eng., Rensselaer Polytechnic Inst., Troy, N. Y.

[32a] Received by the Secretary July 28, 1934.

$\frac{x_1}{l_1} = 0.6$ and $\frac{x_1}{l_1} = 0.7$, were negative and occurred for an advancing live load in the main span at the highest temperature. The maximum moment at the middle of the main span was positive, but occurred for an advancing live load in that span at highest temperature. The chart calls for a part of the main span loaded symmetrically about the center line at highest temperature. It must be admitted that the difference between the maximum functions at the center of the main span, obtained from the two different conditions of loading, was small.

As stated in the paper, the effect of continuity at the towers is lost in the spans at points proportionately nearer and nearer the towers as the span length increases. Conversely, the opposite is true for decreasing span lengths. In the example, solved by the writer, this is strikingly brought out by the fact that the effect of continuity at the towers is felt in the side spans for a distance of two-fifths of the span, whereas, in Mr. Steinman's example, with the span lengths just double, this effect is felt in the side span for a distance of only one-fifth of the span. Perhaps, this accounts for the disagreement with the loading chart at the sections, $\frac{x_1}{l_1} = 0.6$ and $\frac{x_1}{l_1} = 0.7$.

Some may question the advisability of using the deflection theory for the analysis of the stiffening trusses for such short span lengths, feeling that the approximate theory will give results sufficiently accurate for practical purposes. Using a 3 to 1 ratio of dead load to live load, the writer found that the average deflection correction for live load alone amounted to 16%, which seems to justify the use of the deflection theory formulas even for comparatively short spans.

While, unquestionably, the great worth of this paper lies in its presentation of a workable theory for hingeless and multiple-span suspension bridges, the ease with which the deflection theory may be applied in the solution of two-hinged stiffening trusses, is worthy of note. This feature alone should commend it to the profession. The writer finds that by using the simplified equations of the paper, it is possible to obtain a solution for a two-hinged stiffening truss by the deflection theory almost as rapidly as by the approximate theory. The paper demonstrates a skill in condensing and simplifying difficult algebraic equations, and describes a complicated theory in a form both easy to understand and to apply.

A. W. Fischer,[33] Esq. (by letter)[33a].—Remarkable ingenuity is demonstrated by the derivation of the general formulas in this paper. These formulas can be used for continuous stiffening trusses—and for non-continuous suspension bridges—by simply writing zero for T wherever it occurs. If hyperbolic functions are used, formulas can be derived so that the moments and shears can be solved in less time than it requires to solve the formulas as given by the author.

---

[33] Philadelphia, Pa.

[33a] Received by the Secretary August 4, 1933.

Using the same notation and the differential equation given by the author:

$$\frac{d^2\eta}{dx^2} - c^2\eta = -\frac{1}{EI}(M_0 - H y + T) \ldots\ldots\ldots\ldots (72)$$

Equation (72) is a linear differential equation with constant coefficients, the second member of which is a function of x. The complete integral of this formula consists of two parts, a complementary function and a particular integral, the complementary function being the complete solution of the equation formed by making the first member of Equation (72) equal to zero.[34]

The solution of the particular integral is not difficult, but is somewhat lengthy.[35] The complete solution of Equation (72), is:

$$\eta = C_1 \sinh cx + C_2 \cosh cx - \frac{1}{EIc^2}\left[\left(\frac{p_x}{c^2} - \frac{H}{rc^2}\right) - M_0 + H y - T\right] . (73)$$

which is similar to Equation (7) of the paper, except that hyperbolic functions enter into it.

The second derivative of Equation (73) gives the value of moment,

$$M = -(H_w + H)(C_1 \sinh cx + C_2 \cosh cx) - \frac{H}{rc^2} + \frac{p_x}{c^2} \ldots (74)$$

and the third derivative gives the value of shear,

$$V = -(c)(H_w + H)(C_1 \cosh cx + C_2 \sinh cx) \ldots\ldots\ldots (75)$$

For a uniform load of p_x per unit of length extending a distance, k, from the left end of the span (which is Case III of the paper) the constants of integration for the loaded section, *AB*, and the unloaded section, *BC*, may be determined at the same time. The constants for the loaded sections will be denoted by C_1 and C_2; those for the unloaded part by C_3 and C_4. The additional formulas now required for the four constants are obtained from the condition that the moments and shears at the right end of Section *AB* are equal to those at the left end of Section *BC*.

From Equations (74) and (75), the moments and shear for the various sections are:

$$M_{AB} = -(H_w + H)(C_1 \sinh cx + C_2 \cosh cx) - \frac{H}{rc^2} + \frac{p_x}{c^2} \ldots (76a)$$

and,

$$M_{BC} = -(H_w + H)(C_3 \sinh cx + C_4 \cosh cx) - \frac{H}{rc^2} + 0 \ldots (76b)$$

$$V_{AB} = -c(H_w + H)(C_1 \cosh cx + C_2 \sinh cx) \ldots\ldots (77a)$$

and,

$$V_{BC} = -c(H_w + H)(C_3 \cosh cx + C_4 \sinh cx) \ldots\ldots (77b)$$

From Equations (76) and (77), and the conditions that when $x = 0$, $M = T_1$ in Equation (76a) and for $x = l$, $M = T_2$ in Equation (76b) and from the conditions stated previously—the values of the constants can be determined. Solving for C_1:

---

[34] "Differential Equations", by Abraham Cohen, First Edition, p. 96.

[35] "Differential Equations", by Daniel A. Murray, Eighth Impression, p. 72.

$$C_1 = \frac{1}{(H_w + H)\sinh c\,l}\left[\frac{H}{r\,c^2}(\cosh c\,l - 1) - \frac{p_x}{c^2}(\cosh c\,l - \cosh c\,k \cosh c\,l + \sinh c\,l \sinh c\,k) + T_1 \cosh c\,l - T_2\right] \quad\ldots\ldots\ldots\ldots(78)$$

and,

$$C_2 = \frac{1}{H_w + H}\left[\frac{p_x}{c^2} - \frac{H}{r\,c^2} - T_1\right] \quad\ldots\ldots\ldots\ldots(79)$$

Substituting the values of C_1 and C_2 in Equation (76a) and reducing, an expression is derived for the moment between the points, A and B, as follows:

$$M_{AB} = \left[\begin{array}{l} + \dfrac{p_x}{c^2}\left\{1 - \dfrac{\sinh c\,x \cosh c\,(l - k) + \sinh c\,(l - x)}{\sinh c\,l}\right\} \\ - \dfrac{H}{r\,c^2}\left\{1 - \dfrac{\sinh c\,x + \sinh c\,(l - x)}{\sinh c\,l}\right\} \\ + \dfrac{T_1 \sinh c\,(l - x)}{\sinh c\,l} + \dfrac{T_2 \sinh c\,x}{\sinh c\,l} \end{array}\right] \quad\ldots\ldots(80)$$

Differentiating Equation (80), an expression is derived for shear in the main span, which gives:

$$V_{AB} = \left[\begin{array}{l} - \dfrac{p_x}{c}\left\{\dfrac{\cosh c\,x \cosh c\,(l - k) - \cosh c\,(l - x)}{\sinh c\,l}\right\} \\ + \dfrac{H}{r\,c}\left\{\dfrac{\cosh c\,x - \cosh c\,(l - x)}{\sinh c\,l}\right\} \\ - \dfrac{c\,T_1 \cosh c\,(l - x)}{\sinh c\,l} + \dfrac{c\,T_2 \cosh c\,x}{\sinh c\,l} \end{array}\right] \quad\ldots\ldots(81)$$

The expression for moments and shears seems lengthy, but if a table of hyperbolic functions and a 20-in. slide-rule is used, the values can be found in about one-half the time it requires to compute the moments and shears by the formulas as given by the author. On close examination it can be seen that the moment using hyperbolic functions can be solved in less time than is required for the constant, C_1, as given by the author. A 20-in. slide-rule does not give reliable results using the author's formulas for Case III loading.

For a given load on the main span as shown for Case III loading the location of the greatest bending moment can be determined by the method of finding the maximum of any continuous function. At the point of greatest bending moment, the differential of the moment, or the shear, must be equal to zero. This results in,

$$\sinh cx = \frac{1}{\left\{\left[\dfrac{\left(\dfrac{H}{r} - p_x + c^2\,T_1\right)\sinh\, c\,l}{p_x \cosh c(l-k) - \dfrac{H}{r} - \left(p_x - \dfrac{H}{r}\right)\cosh c\,l + c^2\,T_1 \cosh c\,l - c^2\,T_2}\right]^2 - 1\right\}^{\frac{1}{2}}} \quad .(82$$

as the formula of the condition for greatest moment, for a certain value of k. From Equation (82) x can be solved readily. Furthermore, by means of Equation (82), the point of maximum moment for an approaching load on the main span can be determined and, in this way, maximum positive moments can be calculated for the main span from the end to near the center. The approaching load can be increased for various values of k until the maximum positive moment is found.

Example 1.—As an example take the author's values from Table 3 when $k = 0.475\,l$, to solve the value of x where the moment is greatest for a load on the main span extending from A to B (Case III loading); thus: $\sinh c\,l = \sinh 6.728 = 417.7$; $\cosh c\,(l - k) = \cosh 3.531 = 17.09$; and $\cosh c\,l = \cosh 6.728 = 417.7$.

The values of the hyperbolic functions are taken from a table in the writer's possession,[36] but for work of this nature a more complete table should be used.[37]

Substituting the foregoing values in Equation (82) and reducing:

$$\sinh c\,x = \frac{1}{\left\{\left[\dfrac{-\,606.0}{-\,585.0261}\right]^2 - 1\right\}^{\frac{1}{2}}} = \frac{1}{\{1.073 - 1\}^{\frac{1}{2}}} = \frac{1}{0.2701} = 3.705$$

Therefore, $cx = 2.02$, and $x = \dfrac{2.02}{0.00841} = 240.4$ ft. The author uses a value of 240 ft.

Equation (82) seems long, but it is easier to calculate cx from this formula than to use the "cut-and-try" system of locating the point of maximum moment for a given load on the main span from AB.

Example 2.—Using the same values as in Example 1, with $x = 0.3\,l$, to solve for the moment, M, in Case III when the load extents from A to B: $\sinh cx = \sinh 2.019 = 3.699$; $\cosh c\,(l - k) = \cosh 3.531 = 17.09$; $\sinh c\,(l - x) = \sinh 4.709 = 55.47$; and $\sinh c\,l = \sinh 6.728 = 417.7$.

Substituting in Equation (80):

$$M_{AB} = \frac{1.300}{0.00007064}\,(1 - 0.2843) - 5\,165\,(1 - 0.1416) - 953$$
$$= 13\,170 - 4\,435 - 953 = +\,7\,782 \text{ ft-kips}$$

The author gives + 7 770 ft-kips. For a two-hinged stiffening truss with the same loading condition, the moment would be 13 170 − 4 435 = + 8 735 ft-kips. Equations (78) to (82), inclusive, are true for a two-hinged stiffening truss, if the terms containing T_1 and T_2 are zero. The solutions of T_1 and T_2 as given by the author are quite long and hyperbolic functions can be used to advantage.

---

[36] Mechanical Engineers' Handbook, by Lionel S. Marks.

[37] See Smithsonian Tables of Hyperbolic Functions.

In view of the simple formulas that can be developed using hyperbolic functions for the solution of moments and shears the writer believes that after formulas are written for the various loading conditions, they will be used in nearly all cases for the solution of moments and shears for continuous or non-continuous suspension bridge stiffening trusses.

As suspension bridges will be used more and more for either long or short spans, the simplest, shortest, and yet most exact analysis should be used.

J. M. FRANKLAND,[38] ESQ. (by letter)[38a].—Ingenuity was shown in arranging the complicated working formulas of this paper so that they assume the most convenient forms for computation. The use of a single function, *d*, from which all the hyperbolic-function terms may be derived, and the publication of a table of its values (Table 1), are of much practical advantage.

It should be noted, however, that, to maintain accuracy in the use of d in its various combinations, the values of d itself should be known to a higher degree of accuracy, to five figures at least. Linear interpolation for intermediate values of $c\,l$ in the published table is not sufficiently accurate at low values. Therefore, the following interpolation formula is suggested:

$$\Delta d = \frac{(1 - d_0)\,(1 + d_0)}{2 + d_0\,\Delta(c\,l)}\,\Delta(c\,l) \qquad \text{(83)}$$

in which, Δd is the increase in d due to an increase in $c\,l$ equal to $\Delta\,(c\,l)$, and d_0 is the value of d from which the increase is reckoned. The values obtained are accurate throughout Table 1 to the nearest figure in the fifth decimal place.

Numerous objections have been raised from time to time with regard to the conventional deflection-theory derivation (that is, as given in Article 7) of the equation for the component of cable tension due to live load. The author[39] himself has written.

"It should be noticed that two approximations are involved in the foregoing [conventional] derivation of the H-equation * * * it is assumed that the suspender and cable loading is uniformly distributed over the span; this is contrary to the actual condition * * *. The second approximation consists in writing the original cable sag (f) instead of the augmented cable sag ($f + \eta$) in the expression for W_1 * * *".

A further objection can be made to the calculation of the work of deformation in terms of the mean forces acting during the displacement, since in a suspension bridge the forces are not linear functions of the displacement.

A kinematical analysis shows that Equation (22), the usual formula for H, is more accurate than the assumptions involved in its derivation and that, in fact, the foregoing objections do not apply to the final equation when the towers are hinged at the base. This analysis may be briefly sketched as follows: The cable under dead load and normal temperature is assumed to have the form of a parabola. Let ϕ be the angle to the original cable curve and the x-axis (assumed horizontal). When the temperature alters and live load is applied, the point, (x, y), of the cable curve is displaced to (x', y'), and ϕ changes to ϕ'. Let,

---

[38] Cons. Physicist, New York, N. Y.

[38a] Received by the Secretary August 9, 1934.

[39] "Modern Suspension Bridges", by D. B. Steinman, M. Am. Soc. C. E., Second Edition, p. 252, New York, John Wiley and Sons, 1929.

$$\eta = y' - y \quad \text{(84)}$$

and,

$$\Delta\phi = \phi' - \phi \quad \text{(85)}$$

If one neglects the change in vertical dimensions of towers and suspenders due to alteration of temperature and stress, as is customary, the stiffening truss has the same deflection, η, as the cable.

If ds is the element of arc in the original cable curve, and ds', the corresponding element of arc in the strained cable,

$$ds' = (1 + \varepsilon)\, ds \quad \text{(86)}$$

in which,

$$\varepsilon = \frac{H \sec \phi'}{E_c\, A_c} \pm \omega\, t \quad \text{(87)}$$

The horizontal projection of the element of strained cable (correct to quantities of the second order) is:

$$\begin{aligned} ds' \cos \phi' &= (1 + \varepsilon)\, ds \cos (\phi + \Delta\phi) \\ &= [(1 + \varepsilon) \cos \phi - \Delta\, \phi \sin \phi]\, ds \quad \text{(88)} \end{aligned}$$

If the cable span, l, increases by Δl,

$$\begin{aligned} l + \Delta\, l &= \int_0^l [(1 + \epsilon) \cos \phi - \Delta\, \phi \sin \phi] \frac{ds}{dx}\, dx \\ &= \int_0^l (1 + \epsilon - \Delta\, \phi \tan \phi)\, d\, x \quad \text{(89)} \end{aligned}$$

and,

$$\Delta l = \int_0^l (\varepsilon - \Delta\, \phi \tan \phi)\, dx \quad \text{(90)}$$

Next, a value of $\Delta\phi$ is wanted in terms of known quantities:

$$\frac{dy'}{ds'} = \sin \phi' = \sin \phi + \Delta\phi \cos \phi \quad \text{(91)}$$

Also,

$$\frac{dy'}{ds'} = \frac{1}{1 + \epsilon} \frac{dy'}{ds} = \frac{1}{1 + \epsilon} \left(\frac{dy}{ds} + \frac{d\eta}{dx} \frac{dx}{ds} \right) = \frac{1}{1 + \epsilon} \left(\sin \phi + \frac{d\eta}{dx} \cos \phi\right) \quad \text{(92)}$$

Equating the two values of $\frac{dy'}{ds'}$ and solving,

$$\Delta\phi = \frac{d\eta}{dx} - \varepsilon \tan \phi \quad \text{(93)}$$

Hence, $\varepsilon - \Delta\phi \tan\phi = \varepsilon \sec^2\phi - \frac{d\eta}{dx}\frac{dy}{dx}$; and,

$$\Delta l = \int_0^l \epsilon \sec^2\phi \, dx - \int_0^l \frac{d\eta}{dx}\frac{dy}{dx}\, dx \qquad (94)$$

It is sufficiently accurate to replace ϕ' by ϕ in the expression for ε. The first integral then becomes:

$$\int_0^l \varepsilon \sec^2\phi \, dx = \frac{H}{E_c A_0} l_s \pm \omega\, t\, l_t \qquad (95)$$

in which,

$$l_s = \int_0^l \frac{A_0}{A_c}\left(\frac{ds}{dx}\right)^3 dx \qquad (96)$$

and,

$$l_t = \int_0^l \left(\frac{ds}{dx}\right)^2 dx \qquad (97)$$

On integrating by parts, the second integral gives:

$$\int_0^l \frac{d\eta}{dx}\frac{dy}{dx}\, dx = \frac{8f}{l^2}\int_0^l \eta \, dx \qquad (98)$$

Thus, the increase in cable span is given by:

$$\Delta l = \frac{H}{E_c A_0} l_s \pm \omega\, t\, l_t - \frac{8f}{l^2}\int_0^l \eta\, dx \qquad (99)$$

Summing from anchorage to anchorage, $\Sigma \Delta l = 0$; $\Sigma l_s = L_s$; and, $\Sigma l_t = L_t$.

In the case of hinged towers with the same value of H in all spans,

$$\sum \frac{8f}{l^2}\int_0^l \eta\, dx = \frac{H}{E_c A_0} L_s \pm \omega\, L_t \qquad (100)$$

which is identical with Equation (22). When the towers are fixed, it will be satisfactory in ordinary cases to use Equation (100) and interpret H as the mean value for the bridge, giving the side spans the weight one-half.

The derivation is not particularly complicated by dispensing with the assumption that vertical dimensions are unchanged. The effect of temperature is greater than that of suspender elongation under stress combined with the shortening in the towers. Considering only the temperature effect on vertical dimensions and taking the same coefficient of thermal expansion for both towers and cables, Equations (99) and (100) become:

$$\Delta l = \frac{H}{E_c A_0} l_s \pm \omega\, t\, l - \frac{8f}{l^2}\int_0^l \eta\, dx \qquad (101)$$

and,

$$\sum \frac{8f}{l^2}\int_0^l \eta\, dx = \frac{H}{E_c A_0} L_s \pm \omega\, t\, L \qquad (102)$$

in which, L is the distance between anchorages. There is little difference between the values of H given by Equations (100) and (102), but the latter is a little simpler, since there is no need to calculate the integral, L_t. To take into consideration the effect of elongation of suspenders under stress gives one a negligible gain in accuracy at the expense of complication of the formulas and is not to be recommended.

After obtaining the foregoing derivation independently, the writer found that Professor G. G. Krivoshein[40] had previously made an equivalent analysis to apply to the case of the Delaware River Bridge.

The most satisfactory method of analyzing suspension bridges with tie-cables, or with fixed towers, is by the use of Equations (101) and (102). The free-hanging tie-cable is completely determined when the change in span is known.

It may not be amiss to point out a corollary to be drawn from the H-equation. The loss of gravitational potential of the dead load is:

$$\sum \int_0^l w\,\eta\,dx = H_w \sum \frac{8f}{l^2} \int_0^l \eta\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (103)$$

At normal temperature Equation (103) is equal (by Equation (102)) to $\dfrac{H\,H_w}{E_c\,A_0} L_s$, which is always a positive quantity at normal temperature. This shows that the center of gravity of the dead load drops in every case when live load is applied. Thus, the dead load, as such, does not resist the live load deformation. Of course, the initial tension, H_w, in the cable resists the live load deformation, but this must be viewed as a secondary (not immediate) effect of dead load. Since it is conceivable that other means than dead load alone may be used to obtain H_w (such as connecting suspenders to the stiffening truss in such a manner as to give zero bending moment at an elevated temperature), the writer prefers to call H_w the initial tension rather than the dead load tension.

The paper represents a distinct advance in the design theory of suspension bridges, supplying the engineer for the first time with an accurate means of computing the behavior of definite structures of those types the peculiar advantages of which have become increasingly apparent as experience in this field has developed. The formulas of the deflection theory, however, do not exhibit clearly the relations among the essential variables which the designer has at his control. For this reason, its use is restricted almost entirely to computing the behavior of designs obtained primarily by other methods. This unfortunate condition has hindered the economic development of suspension bridge design. The inadequacy of the elastic theory has been demonstrated clearly; apart from its quantitative inaccuracy, the qualitative picture it furnishes is far from true. It is earnestly to be hoped that, even at the expense of quantitative accuracy, simpler formulas will be developed on the basis of the deflection theory that will demonstrate directly in their form the true functioning of suspension bridges. The problem is not an easy one because there are many unexpected traps for the unwary.

---

[40] "Simplified Calculation of Statically Indeterminate Structures: Appendix", by G. G. Krivoshein, Prague, the Author, 1930.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SAND MIXTURES AND SAND MOVEMENT IN FLUVIAL MODELS

Discussion

By Messrs. R. H. Keays, and F. T. Mavis.

R. H. Keays,[32] M. Am. Soc. C. E. (by letter)[32a].—Flood control of alluvial rivers and the accompanying problems of erosion and deposition have been studied extensively in Europe by the use of hydraulic models, and more recently in the United States. Much useful information has been obtained in this way; but running all through the literature on the subject doubt seems to be expressed as to their practical application to the prototypes.

It is to be inferred that, in problems of erosion and deposition, the results obtained are at best qualitative only. In some cases it is supposed that the phenomena observed in the model may be reversed in the prototype. The difficulties are well explained in various papers published by the Society.

What is needed now, after so many prototypes have been constructed on the basis of experiments with models, is information as to how these theoretical deductions check in practice. Whatever conclusions are drawn, it is to be noted that no extensive violations of the laws governing the natural flow of water in alluvial rivers are permissible. Some of these phenomena that might be called laws, may be mentioned offhand.

The first of these laws the writer conceives to be that, other things being equal, the slope of the river tends to be substantially constant at all points throughout its course. At first sight, there might seem to be many violations of this law, but closer examination will probably show that other things are not equal, or that recent geological changes have interfered with the river finding its true regimen.

The writer has seen this law stated a little differently, in that all river slopes tend to become flat as one goes down stream. This is usually true, of course, but in this case one speaks of rivers that continually increase in volume down stream by the addition of water from tributaries, and, therefore, other things are not equal.

Note.—The paper by Hans Kramer, Assoc. M. Am. Soc. C. E., was published in April, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by Messrs. John Leighly, Paul W. Thompson, and Gerard H. Matthes.

[32] New York, N. Y.

[32a] Received by the Secretary August 1, 1934.

On the other hand, the Nile is said to have a constant slope from Assuan to the sea. As is well known the Nile has no tributaries in this stretch, and there is even a loss from evaporation. The foregoing statement is particularly true up river from Cairo several hundred miles. The two branches of the river in the extensive delta have a somewhat greater slope.

There are also natural irregularities in the slope of river caused by the influence of large tributaries, especially those of steeper grade carrying much "geschiebe". Such material tends to deposit below the junction with the main river on account of the decrease in velocity. The main river then backs up enough to erode this material, thus flattening the slope up stream.

A second law might be stated by saying that all alluvial rivers tend to flow in a single channel. Islands are a temporary phenomenon from a long-time point of view. They are constantly being formed by cut-offs, but their back channels silt up again.

The writer scarcely dares to suggest a third law because it is such a controversial matter, namely, that all alluvial rivers tend to meander. In general, large rivers tend to form bigger meanders than small ones, but specifically each river is a law unto itself in this respect. Among factors governing the meanders may be mentioned the size of the river, the quantity and character of "geschiebe" carried, relative high and low-water stages, and frequency and regularity of floods. Particularly to be mentioned in this latter respect is the regular annual flood of the Nile as compared to the usual idea of excessive floods of varying intensity at long intervals.

The writer would suggest also that it might be stated as a law, that all alluvial rivers tend to silt up their beds during prolonged low-water periods and to flush them at time of flood. A flood coming after a long period of low-water flow is, therefore, more disastrous than it would be otherwise. On account of this tendency of rivers to silt up their beds during low water, and *vice versa,* it is difficult to establish any flow curves for a river that will indicate the flow merely by observing the stage. Such curves must be checked frequently.

When left to themselves, rivers continually change their channels. The concave beds are extended by erosion, thus lengthening the river and flattening the slope and decreasing the average velocity. Then, a cut-off is made which shortens the river again and restores the original slope.

For many years the policy of the Mississippi River Commission has been to prevent cut-offs, in consequence of which the river has been lengthened about 5 per cent. The writer does not recall, however, that any one has mentioned this fact in proposing new cut-offs in the Mississippi. It would be quite reasonable to suppose that the river might be shortened as much as it has been lengthened without causing any trouble.

F. T. MAVIS,[33] ASSOC. M. AM. SOC. C. E. (by letter)[33a].—Data presented in this paper should be helpful in the proportioning of river models with erodible beds. Table 1, showing in detail the data obtained in the tests, is a contribution to the study of movable bed models. The writer feels, however, that the sands used in the tests have been inadequately described by the gradation curves plotted on Cartesion co-ordinate paper in Fig. 1. Gradation curves plotted on semi-logarithmic paper, or on logarithmic-probability paper, give a more accurate picture of the gradation of sands over the entire range from fine to coarse-grained sizes.

---

[33] Assoc. Prof., Mechanics and Hydraulics, The State Univ. of Iowa, and Assoc. Director in Charge of Laboratory, Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[33a] Received by the Secretary August 2, 1934.

The author recognizes three variables as affecting the results of his experiments, namely, depth, slope, and sand mixture. He states that "except for special isolated cases, the study of velocity was purposely not undertaken because that is a derived and not an independent criterion, being a function of depth and slope." Evidently, the author is tacitly making further assumptions: (1) That cross-currents are negligible; (2) that velocity distributions (that is, velocity-depth curves) are virtually identical at each cross-section of the channel; and (3) that the shape and specific gravity of sand particles are the same for each mixture. Apparently, he is using the Chezy formula, $V = C \sqrt{RS}$, to express the relationship between velocity, depth, and slope.

If the coefficient, C, in the Chezy formula was actually a constant it would, of course, be independent of Reynolds' number, $\mathsf{R} = \frac{V\,d}{\nu}$. Experience seems to indicate that, for a given channel, C varies approximately as $\mathsf{R}^{\frac{1}{2\,m}}$, in which, $m = 1$ in the range of laminar flow, and $m = 4$ to 6, or more, in the range of turbulent flow. If the coefficient of kinematic viscosity, ν, is constant this leads at once to the well-known general expression:

$$V = k\,R^{ma+a}\,S^{ma} \qquad \text{(12)}$$

in which, $a = \frac{1}{2\,m - 1}$. If $m = 5$, and if k is a function of the roughness of the channel and the coefficient of kinematic viscosity,

$$V = k\,R^{\frac{2}{3}}\,S^{\frac{5}{9}} \qquad \text{(13)}$$

Inasmuch as the slope in a model is equal to the vertical scale distortion factor times the corresponding slope in the prototype—if the model is constructed to scale—it will be evident that, with small error, Equation (13) can be replaced by the Manning formula:

$$V = \frac{1.486}{n}\,R^{\frac{2}{3}}\,S^{\frac{1}{2}} \qquad \text{(14)}$$

in which, n is the roughness factor.

The writer computed values of Manning's n from the data given in Table 1 and plotted curves (of which Fig. 16 for Series I is an example) for each series, slope, and hydraulic radius. These curves seem to justify the following observations relative to the author's tests:

(1) Within the "zone of usefulness", as defined by the author, the median value of Manning's *n* for each series is, as follows:

Sand	Size of grains, in millimeters	Median value of Manning's *n*
I	0 to 5.00	0.024
II	0 to 1.77	0.022
III	0.385 to 5.00	0.025

One-half the number of observations in this zone deviate less than about 8% from these values of *n*.

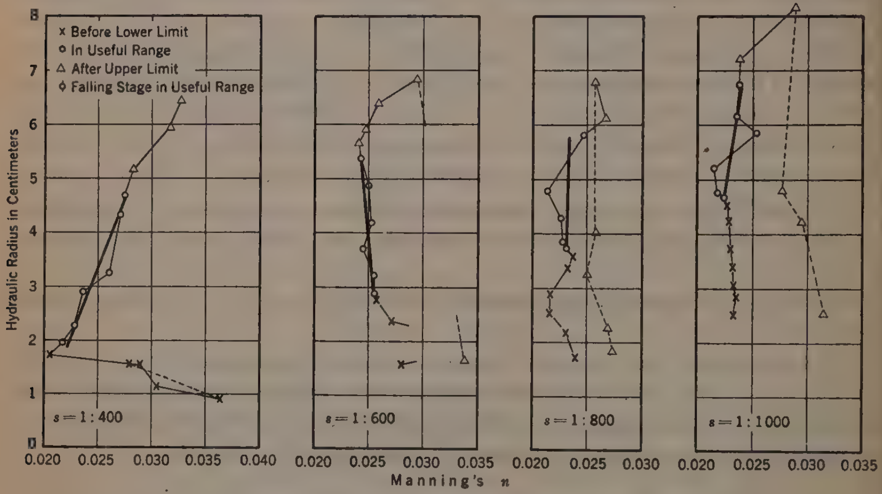

FIG. 16.—ROUGHNESS COEFFICIENTS FOR RIVER MODELS, SERIES I.

(2) After riffles have formed, that is, above the "upper limit", Manning's *n* generally increased sharply to values of the order of 20% above values in the zone of usefulness.

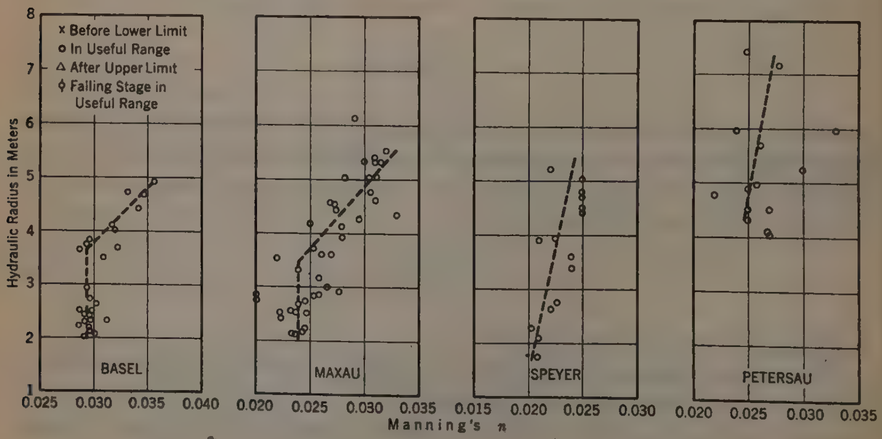

FIG. 17.—ROUGHNESS COEFFICIENTS FOR THE RHINE RIVER.

(3) Before general bed movement takes place in channels where the depth of flow is greater than 1.5 cm, values of Manning's n are essentially the same as, or somewhat less than, corresponding values in the zone of usefulness.

Fig. 17 was prepared from data collected by Heinrich Wittmann[34] in a study of the effect of correction works between Basel, Switzerland, and Mannheim, Germany, upon the bed-load movement of the River Rhine. These observations on the Rhine indicate that there is a general increase in the value of Manning's n with an increase in bed-load movement, and they would not appear to be at variance with the observations reported by the author in Table 1.

---

[34] "Der Einfluss der Korrektion des Rheins Zwischen Basle und Mannheim auf die Geschiebebewegung des Rheins", von Heinrich Wittmann, *Deutsche Wasserwirtschaft*, Heft 10, 11, 12. (Doctor's dissertation prepared under the direction of Prof. Rehbock and Dr. Boss, Karlsruhe, Germany.)

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

LABORATORY TESTS OF MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGES

Discussion

BY CARROLL L. MANN, JR., ESQ.

CARROLL L. MANN, JR.,[9] ESQ. (by letter)[9a].—Following parallel with the investigation of concrete arches at the University of Illinois and during the first half of 1933, the writer, working under the direction of George E. Beggs, M. Am. Soc. C. E., built and analyzed by the deformeter gauge method,[10] a precise celluloid model of a multiple-span arch bridge.[11]

The word, "precise", as applied to these models, has a significant meaning. Throughout the entire analysis of the model various experiments were performed on seemingly unimportant details to determine the best methods of construction and operation. The work, therefore, served a secondary purpose in that the results of these observations were recorded as specifications for the best possible technique of precise model construction and operation.

Model of Three-Span Arch Bridge.—The continuous concrete structure, the model of which is shown in Fig. 23, has three arch ribs each of 27-ft span, 6.75-ft rise, and is supported by two abutments and two 20-ft piers. Both the abutments and the two pier bases are assumed as fixed, and, consequently, the structure is indeterminate in the ninth degree. For the analysis of this model, a floating gauge was attached and carefully oriented at the crown section of each of the three ribs and the components of crown reactions were determined at these sections as the 1 000-lb vertical unit load was assumed to move across the structure. There were eight load points on each concrete arch rib, spaced 3 ft apart along the horizontal.

NOTE.—The paper by Wilbur M. Wilson, M. Am. Soc. C. E., was presented at the Joint Meeting of the Structural Division, Am. Soc. C. E., and the Applied Mechanics Division, Am. Soc. M. E., Chicago, Ill., June 29, 1933, and was published in April, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by C. B. McCullough, M. Am. Soc. C. E.

[9] Insp., Lock and Dam No. 3, Fayetteville, N. C., Dist. Office, U. S. Corps of Engrs., Wilmington, N. C.

[9a] Received by the Secretary July 16, 1934.

[10] For explanation of the method, see *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 1208–1230; also, McCullough and Thayer, "Elastic Arch Bridges", Chapter VII.

[11] "Model Analysis of Multiple-Span Concrete Arch Bridge", by Carroll L. Mann, Jr., Thesis presented to Princeton Univ. in partial fulfillment of the requirement for the degree of Civil Engineer.

With the components of the forces experimentally determined at each of the three crown sections, the structure became statically determinate. These forces were then transferred to the abutments and pier bases by means of statics and the terminal reaction influence lines so derived.

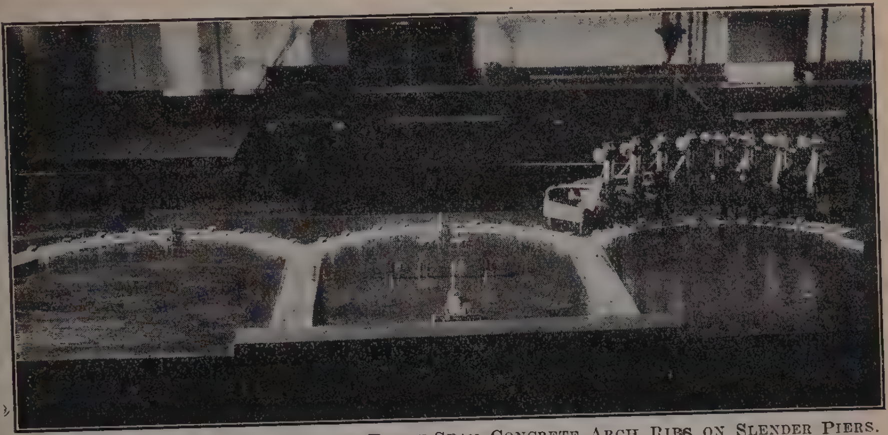

FIG. 23.—CELLULOID MODEL OF THE THREE-SPAN CONCRETE ARCH RIBS ON SLENDER PIERS.

A clearer detail of the right-hand span of the multiple-arch model, including the microscopes, the floating gauge, and the anchorage of the arch terminal between steel plates, is shown in Fig. 24.

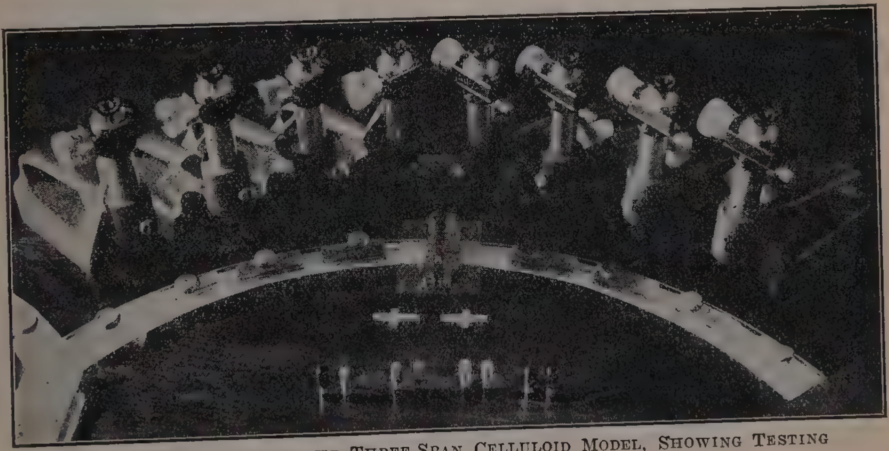

FIG. 24.—END SPAN OF THE THREE-SPAN CELLULOID MODEL, SHOWING TESTING INSTRUMENTS.

Comparison of Experimental and Theoretical Results.—Influence lines for horizontal, vertical, and moment reactions at the left abutment, *A*, of the three-span structure, for a 1 000-lb unit load, are shown in Fig. 25. The circles represent the results obtained for the concrete structure by weighing the reaction components on very precise lever scales. The thrust lines for dead load, as computed both from the model influence values and by theory,

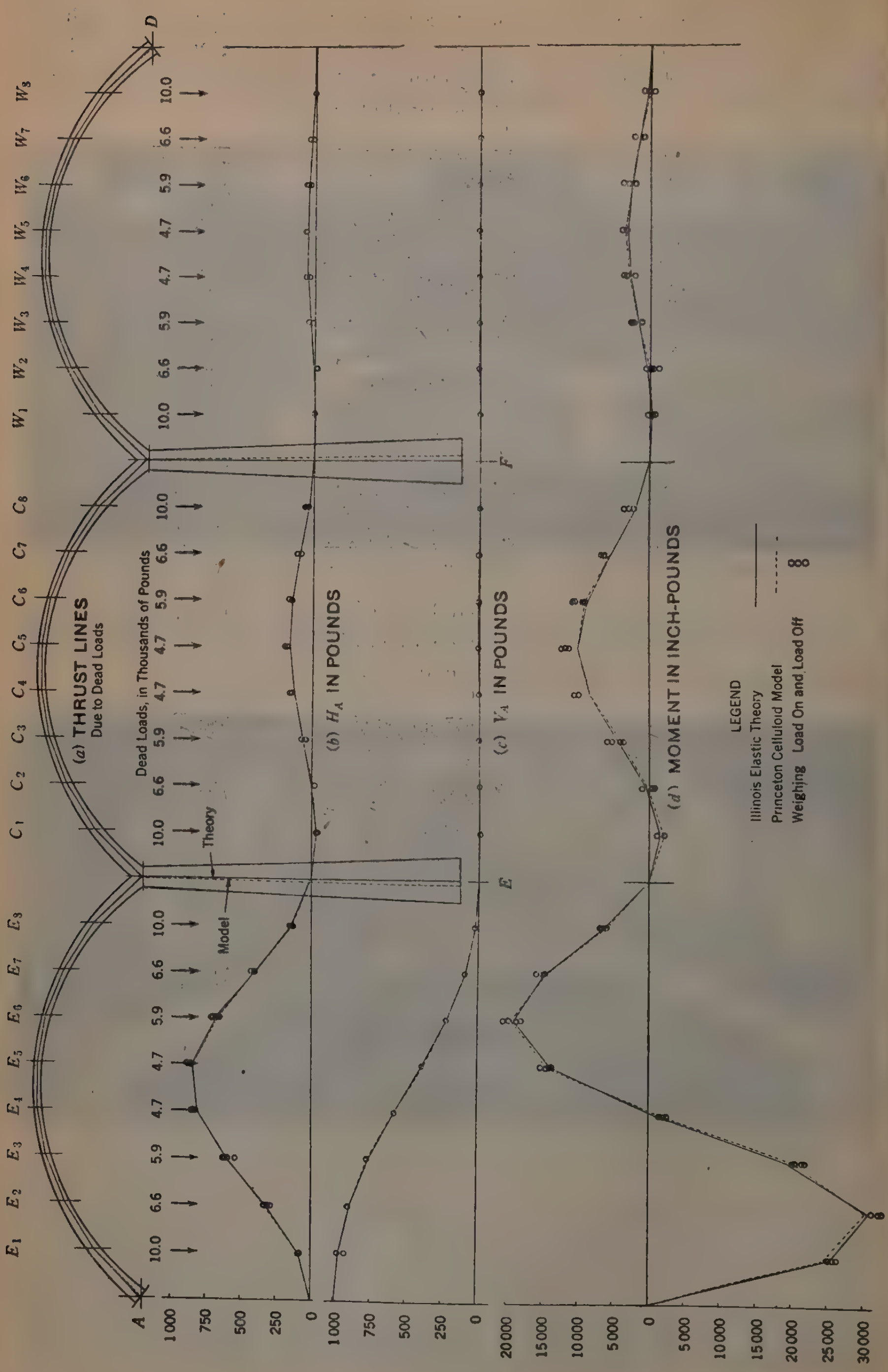

FIG. 25.—INFLUENCE LINES FOR REACTIONS AT A DUE TO 1 000-POUND UNIT LOAD.

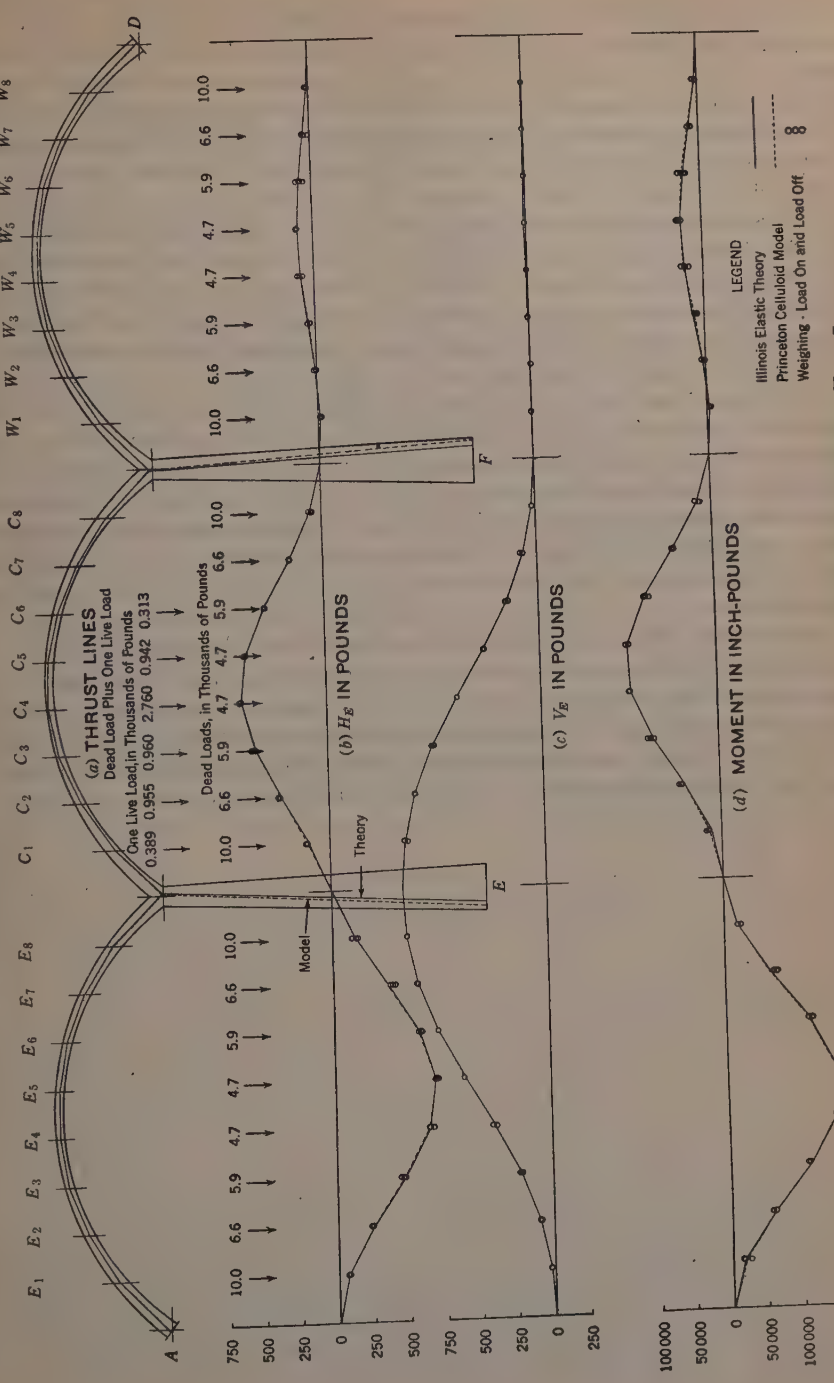

FIG. 26.—INFLUENCE LINES FOR REACTIONS AT E DUE TO 1 000-POUND UNIT LOAD.

are shown in Fig. 25(*a*). This structure was shaped so that the thrust line for dead load might follow closely the center line of the arch ribs and piers, passing precisely through the centers of the arch crowns and the terminals. In the theoretical analysis, rib-shortening was taken into account, but not pier-shortening. It will be noted that thrust lines determined by model and by theory are for the most part almost identical in the arches, separating a negligible amount at the end portions. The two lines deviate slightly down the piers, the theory following the center line and the model deviating from it. The influence lines for reactions at the left base, *E,* as determined by the three methods, are shown in Fig. 26. The thrust lines as computed from model and theory for dead load plus one live load are plotted in Fig. 26(*a*). In general, the deviation of the model thrust lines from the theoretical thrust lines is small, both for dead load and for dead load plus one live load.

If Figs. 25 and 26 are compared with Fig. 15 of the paper, it will be noted that the thrust lines determined from the measured reactions on the prototype diverge from the theoretical thrust lines in the same direction as the thrust lines found from the model test. In no case are the deviations from elastic theory of practical importance; but, nevertheless, the agreement of the test of the prototype and of the model in this divergence of the position of thrust lines from the theory may be significant; and it excites some interest.

Conclusion.—The small variations between results obtained by the theory, by the model, and by the test of the concrete structure are not of practical importance. The differences may be due in part to experimental errors, but undoubtedly some of them are chargeable to assumptions made in the theory. The theory necessarily includes more assumptions than the model, in which Nature itself works automatically, with few limitations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

STRESSES IN SPACE STRUCTURES

Discussion

BY MESSRS. WILLIAM R. OSGOOD, L. E. GRINTER, AND CHARLES M. SPOFFORD.

WILLIAM R. OSGOOD,[3] ASSOC. M. AM. SOC. C. E. (by letter).[3a]—Before reading this interesting paper it had never occurred to the writer that the equilibrium of a set of concurrent forces in space could be established by determining the conditions of equilibrium of a set of co-planar forces related to the set in space. Professor Constant is to be thanked for calling the attention of American engineers to the method.

The convenience of the method is so great that a study of the fundamentals from a slightly different point of view may be desirable. The following derivation, which seems simpler than Professor Constant's presentation, may be helpful to others in understanding the method.[4]

Let O in Fig. 1 represent the joint of a structure in space, and let F represent one of a system of forces, F_1, F_2, F_3, ..., F_n, acting at O. Each of these forces may be replaced by a horizontal force (reversed in the diagram):

$$F' = F \cos \phi \tag{6}$$

anywhere in a vertical plane parallel to F, a vertical force,

$$R = F \sin \phi \tag{7}$$

in a plane containing F and with the line of action at such a distance, d, from O that,

$$R d = F' h \tag{8}$$

and a horizontal couple,

$$S_0 = F' b \tag{9}$$

By adjusting the values of d and S_0, the distances, h and b, respectively, may be taken at pleasure. If all the distances, h, measured vertically, are

---

NOTE.—The paper by F. H. Constant, M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[3] Materials Testing Engr., National Bureau of Standards, Washington, D. C.

[3a] Received by the Secretary May 31, 1934.

[4] See, also, "Graphische Statik räumlicher Kräftesysteme", von R. v. Mises, *Zeitschrift für Mathematik und Physik*, Vol. 64, 1917, p. 209.

taken the same for all the forces of the given system, then $F'_1, F'_2, F'_3, \ldots, F'_n$ form a co-planar set; and by choosing, suitably, the distances, $b_1, b_2, b_3, \ldots, b_n$, measured horizontally perpendicular to $F_1, F_2, F_3, \ldots, F_n$, respectively, it is possible to assure equilibrium of the co-planar set. Since the forces, F', are planar projections of the forces, F, which are in equilibrium, the resultant of the forces, F', cannot be a force; and if the condition,

$$\Sigma F' b = 0 \ldots\ldots\ldots\ldots\ldots (10)$$

can be satisfied, the resultant will not be a couple. Equations (9), (6), and (7) give,

$$b = \frac{S_0}{R} \tan \phi \ldots\ldots\ldots\ldots\ldots (11)$$

and this value of b substituted in Equation (10) gives,

$$\Sigma F' \frac{S_0}{R} \tan \phi = 0 \ldots\ldots\ldots\ldots\ldots (12)$$

Now, if S_0 in each case is adjusted so that the ratio, $\frac{S_0}{R} = a =$ constant, for each force, and b is then taken as $b = a \tan \phi$ (Equation (4)), Equation (12) becomes,

$$a \Sigma F' \tan \phi = a \Sigma R = 0 \ldots\ldots\ldots\ldots\ldots (13)$$

Equation (13) is seen immediately to be satisfied since ΣR is the vertical component of the resultant of the given system of forces, $F_1, F_2, F_3, \ldots, F_n$, and these forces are in equilibrium. The set of forces, $F'_1, F'_2, F'_3, \ldots, F'_n$, obtained as indicated now form a co-planar, non-concurrent system in equilibrium which may be solved in any convenient manner. The forces, F, are then obtained from Equation (6).

L. E. GRINTER,[5] ASSOC. M. AM. SOC. C. E. (by letter)[5a].—An ingenious method for reducing the analysis of stresses in space structures to the problem of stress calculation in a plane is offered by Professor Constant. His explanation of this procedure is stated so clearly, in the simplest terms of mechanics, that one should have little difficulty in following the argument. The method should appeal particularly to those practicing engineers who, because of continual association with planar structures, prefer not to think in terms of stresses in space. The method possibly would have less appeal to the young engineer who has been thoroughly grounded in the study of stresses in three dimensions.

The problem of the analysis of a space structure cannot be solved by a standardized procedure in the manner of a simple roof truss or bridge truss. The explanation lies in the fact that there are many more types of space structures than there are types of planar structures. Of the determinate type alone there is the possibility of widely varying arrangements of reactions and of innumerable bar patterns giving rise to a pedestal, tower, dome, or envelope. Perhaps of greatest significance in this connection is the distinction between those types that may be analyzed by simple equations and those that require simultaneous equations.

---

[5] Prof., Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[5a] Received by the Secretary July 6, 1934.

Based on the foregoing distinction, the example analyzed by the author should be classed as a simple type of space structure because simultaneous equations are not involved. Referring to Fig. 2, one will note that at Joint 2, Members 1-2, 2-1′, and 2-2′ all lie in a single plane and, therefore, the component of the stress in Member 2-3 taken perpendicular to Plane 1′-2′-2-1 must equal the component of the 1 000-lb load perpendicular to the same plane, which shows the stress in Bar 2-3 to be—400 lb. At each of Joints 1, 2, 3, and 4, two such simple resolutions of forces make it possible to determine all stresses in the members of the top ring and in the sloping diagonals. With the stresses in the diagonals known, one can calculate the stresses in the members of the bottom ring and the values of the reactions by resolving forces in three directions at Joints 3′, 2′, 4′, and 1′, in succession. Simultaneous equations are not required.

The writer is not certain exactly to what extent this procedure is simplified by the theory of conjugate forces presented by the author. Possibly the main advantage from the common viewpoint would be the reduction of the analysis to a problem in planar graphics as illustrated by Fig. 3. Maxwell's diagram is used so widely by practicing engineers and even by undergraduate students, that the analysis of the space structure by the use of a graphical procedure similar to the Maxwell diagram would be certain to prove attractive.

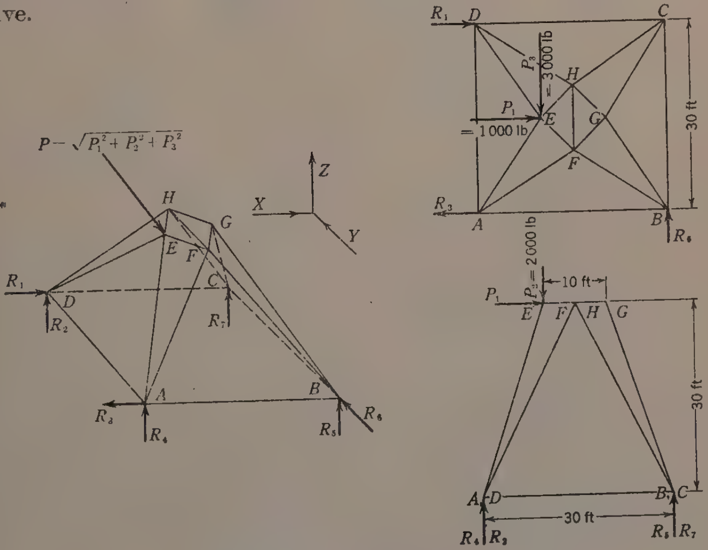

FIG. 4.—LOADS AND DIMENSIONS FOR PEDESTAL.

The writer has been interested in the study of space structures of a more complex type than that illustrated by Fig. 2. The space sketch of Fig. 4 shows a pedestal quite similar to the one analyzed by Professor

Constant, but which is far more difficult to analyze by the procedure used by the writer. It will be noticed that there is no starting point for stress calculations at any joint of the upper ring, because at each joint there are only two unknown stresses that lie in a single plane. At the base it is possible to calculate directly the three horizontal reactions needed for stability, but the four vertical reactions are not determinable except in connection with the bar stresses.

TABLE 1.—SOLUTION OF SIMULTANEOUS EQUATIONS

Equation	*V*	Number	*DH*	*DE*	*DC*	*DA*	*CG*	*CB*	*BF*	*BA*	Equations combined
(1)	1	—3666	$\frac{30}{35}$	$\frac{30}{35}$	......	......	......	......	......	......	(1)
(2)	......	1500	$\frac{15}{35}$	$\frac{10}{35}$	1	......	......	......	......	......	(2)
(3)	......	......	$-\frac{10}{35}$	$-\frac{15}{35}$	......	—1	......	......	......	......	(3)
(4)	—1	1666	$-\frac{30}{35}$	......	......	......	$\frac{30}{35}$	......	......	......	(4)
(5)	......	......	$\frac{15}{35}$	......	—1	......	$-\frac{10}{35}$	......	......	......	(5)
(6)	......	......	$\frac{10}{35}$	......	......	......	$-\frac{15}{35}$	—1	......	......	(6)
(7)	1	......	......	......	......	......	$-\frac{30}{35}$	......	$\frac{30}{35}$	......	(7)
(8)	......	......	......	......	......	......	$\frac{10}{35}$	......	$-\frac{15}{35}$	—1	(8)
(9)	......	3000	......	......	......	......	$-\frac{15}{35}$	1	$\frac{10}{35}$	......	(9)
(10)	......	1500	$\frac{30}{35}$	$\frac{10}{35}$	......	......	$-\frac{10}{35}$	......	......	......	(2) and (5)
(11)	......	3000	$\frac{10}{35}$	......	......	......	$-\frac{30}{35}$	......	$\frac{10}{35}$	......	(6) and (9)
(12)	1	—3666	$\frac{10}{35}$	......	......	—2		......	......	......	(1) and (3)
(13)	1	—8166	$-\frac{60}{35}$	......	......	......	$\frac{30}{35}$	......	......	......	(1) and (10)
(14)	1	......	......	......	......	......	$-\frac{10}{35}$	......	......	—2	(7) and (8)
(15)	1	—9000	$-\frac{30}{35}$	......	......	......	$\frac{60}{35}$	......	......	......	(7) and (11)
(16)	......	—7333	$-\frac{60}{35}$	......	......	......	$\frac{90}{35}$	......	......	......	(4) and (15)
(17)	......	1666	$-\frac{30}{35}$	......	......	......	$\frac{20}{35}$	......	......	—2	(4) and (14)
(18)	......	—6500	$-\frac{90}{35}$	......	......	......	$\frac{60}{35}$	......	......	......	(4) and (13)
(19)	......	—2000	$-\frac{20}{35}$	......	......	—2	$\frac{30}{35}$	......	......	......	(4) and (12)
(20)	......	—10666	......	......	......	......	$\frac{50}{35}$	......	......	4	(16) and (17)
(21)	......	—3000	......	......	......	......	$\frac{50}{35}$	......	......	......	(16) and (18)
(22)	......	—1333	......	......	......	6	......	......	......	......	(16) and (19)
Stresses	4433	......	—1133	233	—1083	222	2100	—1222	—3066	1916	......

A check upon the number of joints, bars, and reactions shows that there is a total of twenty-four reactions and bar stresses and eight joints. Since three equations of statics are available at each joint, the structure is statically determinate. A number of procedures might be adopted for the calculation of stresses, but the following seems to the writer to be as simple as any. The reactions will be calculated first, the value of R_5 from Fig. 4 being given the unknown value, V. The entire structure is treated as a free body; thus $\Sigma F_y = 0$; $R_6 - 3\,000 = 0$; $R_6 = +3\,000$; and, from $\Sigma M = 0$ about R_5 as an axis: $15\,000 - 60\,000 + 30\,R_1 = 0$, hence, $R_1 = +1\,500$ and $R_3 = -2\,500$. Assume that $R_5 = +V$, and then one obtains:

$\Sigma M_{CD} = 0; 30\,000 + 90\,000 - 30\,V - 30\,R_4 = 0; R_4 = 4\,000 - V$

$\Sigma M_{BC} = 0; 30\,000 - 40\,000 + (120\,000 - 30\,V) + 30\,R_2 = 0; R_2 = V - 3\,666$

$\Sigma M_{AD} = 0; 30\,000 + 20\,000 - 30\,V - 30\,R_7 = 0; R_7 = 1\,666 - V$

$\Sigma F_z = 0; -2\,000 + V + (4\,000 - V) + (V - 3\,666) + (1\,666 - V) = 0$ (Check)

From a study of the equations, $\Sigma F_z = 0$ at Joints F, G, and H, it becomes clear that the stresses in the following pairs of members are equal: AF and BF, BG and CG, and CH and DH. There are only eight unknown bar stresses and the unknown reaction, V, at Joints B, C, and D. The nine simultaneous equations that can be written for these joints are sufficient for the evaluation of these unknowns. The proper equations based upon the assumptions that all members are in tension and that the positive directions are upward, backward, and to the right, are given in the first nine lines of Table 1. The right-hand term of each equation is zero. Coefficients are determined from the projected lengths of the members. The solution of these simultaneous equations is accomplished by means of the combined equations given in Table 1, Lines 10 to 22, inclusive, whereas the final stresses

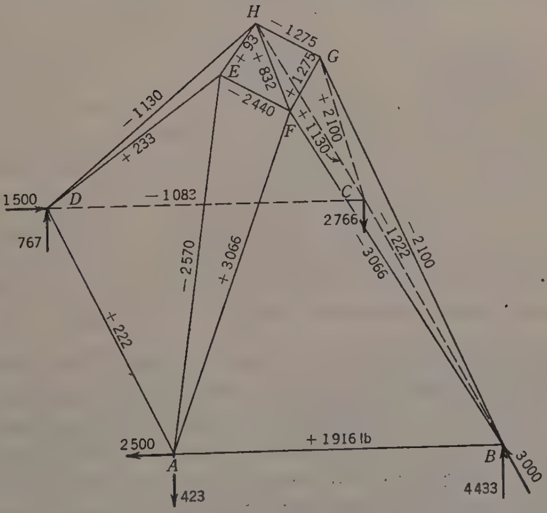

FIG. 5.—STRESSES IN MEMBERS OF PEDESTAL.

are found in the last line of the table. With the stresses from Table 1 known, one can easily determine all other stresses in the pedestal as are shown in Fig. 5. These stresses, computed entirely by use of an 8-in. slide-rule, are not exact, but are quite accurate enough for design purposes.

The analysis of this pedestal by the method of conjugate forces is not as simple as might be expected. Fig. 6 shows a plan view of the structure with the conjugate forces corresponding to Joint *E* in place, based on an arbitrarily selected value of 5 ft for the constant, *a*. Loads P_2 and P_3 from Fig. 4 have been combined into the force, F_2, in order to simplify the conjugate forces. Of course, the six conjugate forces shown form a system of planar forces in equilibrium; but, since there are four unknowns that intersect only in pairs, a direct solution of any one is impossible. Hence, simultaneous equations appear to be necessary.

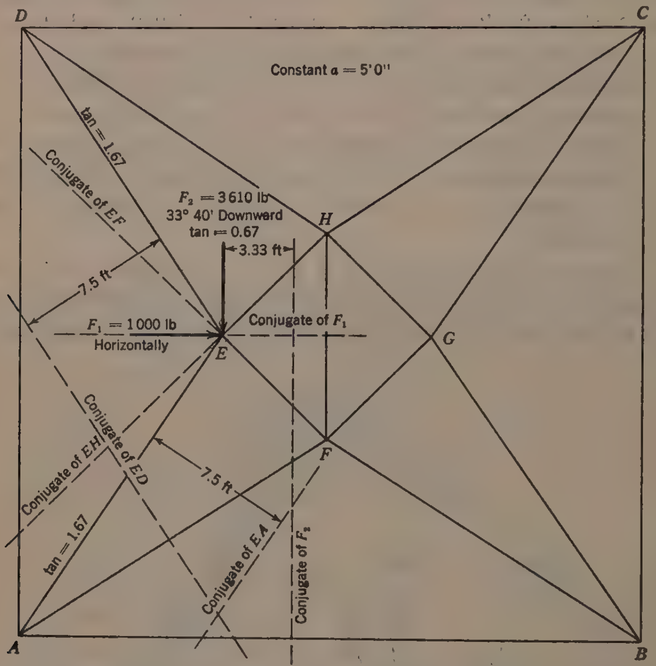

FIG. 6.—PLAN OF PEDESTAL SHOWING CONJUGATES FOR JOINT *E*.

Undoubtedly, one could introduce the conjugate forces for all other joints and then obtain a solution by simultaneous equations, but the writer is not certain as to just how this is best accomplished. In his own calculations, he was able to reduce the analysis to the solution of nine simultaneous equations involving the forces at Joints *B*, *C*, and *D* only. It is clear that a study of Joints *E*, *F*, *G*, and *H* would involve thirteen unknowns and would not permit of a solution by means of the twelve equations available. Hence, in any method it would seem necessary to tie in the analysis with the lower joints of the structure. Professor Constant would perform a service if he would be willing to present his own solution of this problem by the method of conjugate forces. Unless some device such as the use of substitute members can be found that will reduce the number of simultaneous equations to less than nine, the writer doubts that there would be any simpler analysis than the one herein presented. Any one who has attempted the solution of nine simultaneous equations by an ordinary slide-rule would welcome any method of reducing the number of simultaneous equations involved.

The pedestal of Fig. 4 was chosen by the writer for study, because he considers it a practical type of structure for the support of oil-fractionating towers and other such industrial apparatus. It offers advantages over the pedestal analyzed by the author in that all joints involve a symmetrical arrangement of members producing the maximum simplification of members and gusset-plates. There is also the advantage that all compression diagonals are of equal length and as short as possible. Because of the fact that only two members meeting at an upper joint, such as Joint *G*, lie in a single plane, the joint details of such a riveted structure would need careful study, but it is obvious that the pedestal could be constructed quite simply by welding. Where speed of erection is an important factor, the framed steel pedestal should undoubtedly be used in place of the circular concrete chimneys or rigid frame towers that have been widely used for the support of industrial equipment. The high wind stresses and large vertical loads involved could be resisted more economically by a framed pedestal of either the type analyzed by the author, or of the type suggested herein. The explanation is found in the fact that wind force is resisted by direct stress and that the flared pedestal can be given an adequate base without the need for long beam supports at the top. The fact that the major stresses are statically determinate is also an item of importance. Professor Constant has performed a real service by calling the attention of designing engineers to the fact that statically determinate space structures are available for their consideration and that the analysis of these structures need not be particularly complex.

CHARLES M. SPOFFORD,[6] M. AM. SOC. C. E. (by letter)[6a].—An interesting addition to the limited amount of material available in English relating to space frameworks, has been presented by Professor Constant, and the graphical method proposed by him is a convenient means of determining the stresses in such structures. The writer prefers analytical instead of graphical methods for determining stresses in most structures, including space frameworks, inasmuch as such methods are generally quicker and more precise. Moreover, when the computations are properly organized and directed they can be applied to the most complicated structures by inexperienced draftsmen and computers who can perform the necessary numerical work without knowing the underlying principles.

In his conclusions Professor Constant has under-estimated the prevalence of space frameworks in the United States. Common types of such structures are the combination of chords and lateral bracing in curved chord trusses and particularly in continuous bridges and arches, bridge-erection travelers, and water tank towers. The Schwedler dome, which is the most complicated of such structures, is not often built in the United States, although it has been used. Several important domes of this type, however, have been built elsewhere. Transmission towers also frequently require analysis as space frameworks.

---

[6] Hayward Prof. of Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.; Cons. Engr. (Fay, Spofford & Thorndike), Boston, Mass.

[6a] Received by the Secretary July 17, 1934.

It is comparatively simple to carry through the necessary computations by applying the following self-evident theorems[7] which, so far as the writer knows, were first formulated by himself:

Theorem (*a*): If several bars of any framework meet at a joint and all but one lie in the same plane, the component normal to this plane of the stress in that bar which does not lie in the plane will equal the algebraic sum of the components normal to the same plane of all the outer forces which may be applied at the joint under consideration.

Theorem (*b*): The moment of any force or bar stress acting in a given plane about an axis lying in that plane equals zero. It follows that the stress in any bar of a space framework may be determined by the method of moments in the same general manner as in the case of a planar structure; that is, if a section can be taken cutting a number of bars, all of which except bar "a" pass through or are parallel to a given axis, the stress in bar "a" may be found by applying $\Sigma M = 0$ about that axis of all the forces acting on the portion of the structure isolated by the section.

Theorem (*c*): At any joint at which no outer force is applied, and at which the stresses in all bars but two have been found to be zero, the stress in each of these two bars will also be zero, provided the two bars do not lie in the same straight line.

Theorems (*a*) and (*c*) are most useful since, when combined with the equations of statics, they permit the determination (without computations) of the bars in which the stress due to any given load is zero, thus simplifying the computations greatly.

Structures of the Schwedler dome type, chosen by the author for illustration, may or may not be statically determined depending on the pre-establishment of reaction directions. The writer has presented elsewhere[8] a method of investigating the statical condition of such structures. Engineers attempting to solve the ring stresses in one of these structures without first assuring himself that it is statically determined are certain to obtain inconsistent and disappointing results.

It would be helpful if Professor Constant would elaborate the method of conjugate forces to show its application to a Schwedler dome having more than one story and more than four sides.

---

[7] "Theory of Structures," by Charles M. Spofford, M. Am. Soc. C. E., Third Edition, McGraw-Hill Book Co., N. Y., p. 457.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

WAVE PRESSURES ON SEA-WALLS AND BREAKWATERS

Discussion

By Charles E. Fowler, Esq.

Charles E. Fowler,[10] Esq. (by letter)[10a].—The detailed solution of wave pressures on engineering structures is very important and timely, and should lead to more careful designing of them. The use of a factor of safety of 2 is usually ample, as is used in other foundation work, but where the fundamental data are not complete or reliable, a larger factor should be used. The writer has found that Captain Gaillard's formulas[2] are not always representative of the actual facts encountered. Thus, in the Gulf hurricane of 1915, waves higher than 12 ft were generated by the fetch of about 20 miles across Lake Borgne, in Louisiana, and in a similar case at Corpus Christi, Tex., where about the same fetch generated similar waves which destroyed everything in front of them.

The waves of oscillation which occur out in the ocean have little destructive force except as they are encountered by ships plowing through them. When waves reach shallow water, however, they begin to break, and become the destructive waves of translation with which the engineer is concerned in the design of shore structures. Another factor is presented in such cases by storm tides, which in the case of the 1915 hurricane reached 9 ft at Rigolets, La., and 13 ft at the west end of Lake Pontchartrain. Such tides raise the elevation of the point of application of the force of the waves, and thus greatly increase their overturning effect.

The actual maximum wind velocities are usually not easily obtained, but, of course, they are a vital factor of design. Those given in the paper are certainly much lower than those that often occur. For example, in the 1915 hurricane, the maximum velocity reached 140 miles per hr, and while this was only in gusts the sustained velocity was well over 100 miles per hr. The wind velocities on the Pacific Coast, from south of the Columbia

---

Note.—The paper by David A. Molitor, M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[10] Cons. Engr., Lansing, Mich.

[10a] Received by the Secretary July 20, 1934.

[2] *Professional Papers No. 31*, U. S. Corps of Engrs., 1904.

River north to Cape Flattery, often reached 90 miles per hr, and this was the velocity used by the writer in designing the foundation for the Tacoma, Wash., smelter chimney, which was about 600 ft high and which rose to a height of about 700 ft above Puget Sound.

The application of the wave force is at a point above the surface of the water, as shown by the curves accompanying the paper, and these agree well with the curves developed in Italy, and for the harbor work at Halifax, N. S., Canada.[11]

The greatest errors occur in the actual design of the structures built to resist the waves. A vertical surface must resist the full force of the waves, and a sloping surface which has less effective force to resist, often results in other serious problems to be met. In such cases the waves may dash into the air to a height of from 50 to 100 ft, and the tons of water falling from such a height may strike the top of a wall with enough force to destroy (eventually) a masonry wall of concrete, or the fill or structures back of the wall. The curved face of the Galveston sea-wall was designed to throw the water back into the Gulf, which it would do if it was high enough, but a much more scientific method might have been adopted. When the Seattle tide-flats were filled, the front of the fill was protected by a brush bulkhead, 30 ft high, with the butts of the fir brush from 2 to 4 in. in diameter outward, and on a 1:1 slope. In the most severe storms, with a wind velocity of more than 60 miles per hr, the waves would dash against these brush butts, break up completely, and fall back "dead" into the water. The concrete sea wall at North Beach, San Francisco, Calif., had a rough curved exterior which accomplished the same result.[12]

Such designs are only another application of the design of the hydraulic jumps used to kill the force of the water issuing from the impounding or flood-control dams at Dayton, Ohio. The design of other structures to resist wind and wave action should be studied to reduce the exposed surface. In one large bridge structure it was found possible to reduce the exposed surface which had to oppose both wind and waves by a full 50%, by using circular compression members, eye-bar tension members, and lattice floor-beams and stringers. The savage tribes discovered these methods from practical experience as witnessed by the teepee of the American Indians, and in the circular and domed houses of Asiatic towns built to resist the winds and sand storms which blow with great violence.

The mathematical solutions presented in the paper are worthy of the authority presenting them, but too often the young or inexperienced engineer becomes lost in a maze of mathematical formulas, and fails to sense the best practical application of them. Models may be used in laboratory channels or wind tunnels to discover the best forms and as a means of avoiding costly errors. Each problem has its peculiar features, and the solution is often hampered by the materials available, but the best design can always be reached by a careful study of all the features and conditions involved.

---

[11] "World Ports and Harbor Data", by Charles E. Fowler.

[12] "Ideals of Engineering Architecture", by Charles E. Fowler.